Warm-up Problems

1. Draw a graph that matches the given data

\[
\begin{array}{c}
 f'(x) > 0 \\
 f''(x) < 0
\end{array}
\]

2. Draw a graph that matches the given data

\[
\begin{array}{c}
 f'(x) > 0 \\
 f''(x) < 0
\end{array}
\]

3. Draw a graph that matches the given data

\[
\begin{array}{c}
 f'(x) > 0 \text{ when } x < 3 \\
 f'(x) < 0 \text{ when } x > 3 \\
 f''(x) < 0 
\end{array}
\]

4. Draw a graph that matches the given data

\[
\begin{array}{c}
 f'(x) > 0 \text{ when } x < 3 \\
 f'(x) < 0 \text{ when } x > 3 \\
 f''(x) > 0 
\end{array}
\]

Lecture Problems

5. Draw a graph that matches the given data

\[
\begin{array}{c}
 f'(x) > 0 \text{ when } x < 3 \\
 f'(x) < 0 \text{ when } x > 5 \\
 f'(x) > 0 \text{ when } x < 7 \\
 f''(x) < 0 \text{ when } x < 4 \\
 f''(x) > 0 \text{ when } x > 4
\end{array}
\]

6. Draw a graph that matches the given data

\[
\begin{array}{c}
 f'(x) < 0 \text{ when } x < 3 \\
 f'(x) > 0 \text{ when } x > 5 \\
 f'(x) < 0 \text{ when } x < 7 \\
 f''(x) > 0 \text{ when } x < 4 \\
 f''(x) < 0 \text{ when } x > 4
\end{array}
\]
7. Draw a graph that matches the given data

<table>
<thead>
<tr>
<th>$f'(x)$</th>
<th>Condition</th>
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<tbody>
<tr>
<td>$&gt; 0$</td>
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<tr>
<td>$&lt; 0$</td>
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9. Let $f(x) = x^3 - 3x^2 - 9x + 3$

(a) Find all critical points of $f$. **Solution:** $f'(x) = 3(x - 3)(x + 1)$
(b) Find all possible inflection points of $f$. **Solution:** $f''(x) = 6x - 6$
(c) Find all intervals where $f$ is increasing and decreasing.
(d) Find all intervals where $f$ is concave up and down.
(e) Find all local max and mins of $f$.
(f) Find all inflection points of $f$.
(g) Draw a nice graph of $f$. 