Warm-up Problems

1. What is an increasing function?
   (If you don’t know, just make something up that makes sense!)

2. What is a decreasing function?
   (If you don’t know, just make something up that makes sense!)

Lecture Problems

3. For the given functions, find the regions where the function is increasing and decreasing.
   Identify local max and mins.
   (a) $f(x) = x^3 - 9x^2 + 15x + 7$
   (b) $f(x) = x^4 - 8x^3 + 10x^2 + 1$
   (c) $f(x) = 6x^5 - 45x^4 + 50x^3 + 7$

4. Draw a graph that matches the given data

   \[
   \begin{array}{c}
   f'(x) > 0 \\
   f''(x) < 0
   \end{array}
   \]

5. Draw a graph that matches the given data

   \[
   \begin{array}{c}
   f'(x) > 0 \text{ when } x < 3 \\
   f'(x) < 0 \text{ when } x > 3 \\
   f''(x) < 0
   \end{array}
   \]

6. Draw a graph that matches the given data

   \[
   \begin{array}{c}
   f'(x) > 0 \text{ when } x < 3 \\
   f'(x) < 0 \text{ when } x > 5 \\
   f'(x) > 0 \text{ when } x < 7 \\
   f''(x) < 0 \text{ when } x < 4 \\
   f''(x) > 0 \text{ when } x > 4
   \end{array}
   \]

7. Draw a graph that matches the given data

   \[
   \begin{array}{c}
   f'(x) < 0 \text{ when } x < 3 \\
   f'(x) > 0 \text{ when } x > 5 \\
   f'(x) < 0 \text{ when } x < 7 \\
   f''(x) > 0 \text{ when } x < 4 \\
   f''(x) < 0 \text{ when } x > 4
   \end{array}
   \]
8. Draw a graph that matches the given data

\[
\begin{array}{|c|}
\hline
f'(x) > 0 \text{ when } x < 2 \\
f'(x) > 0 \text{ when } 2 < x < 4 \\
f'(x) < 0 \text{ when } 4 < x < 6 \\
f'(x) < 0 \text{ when } x > 6 \\
\hline
f''(x) < 0 \text{ when } x < 2 \\
f''(x) > 0 \text{ when } 2 < x < 3 \\
f''(x) < 0 \text{ when } 3 < x < 5 \\
f''(x) > 0 \text{ when } 5 < x < 6 \\
f''(x) < 0 \text{ when } x > 6 \\
\hline
\end{array}
\]

9. Draw a graph that matches the given data

\[
\begin{array}{|c|}
\hline
f'(x) > 0 \text{ when } x < 2 \\
f'(x) > 0 \text{ when } 2 < x < 4 \\
f'(x) < 0 \text{ when } 4 < x < 6 \\
f'(x) < 0 \text{ when } x > 6 \\
\hline
f''(x) > 0 \text{ when } x < 2 \\
f''(x) > 0 \text{ when } 2 < x < 3 \\
f''(x) > 0 \text{ when } 3 < x < 5 \\
f''(x) < 0 \text{ when } 5 < x < 6 \\
f''(x) > 0 \text{ when } x > 6 \\
\hline
\end{array}
\]

10. Let \( f(x) = x^3 - 3x^2 - 9x + 3 \)

(a) Find all critical points of \( f \).
(b) Find all possible inflection points of \( f \).
(c) Find all intervals where \( f \) is increasing and decreasing.
(d) Find all intervals where \( f \) is concave up and down.
(e) Find all local max and mins of \( f \).
(f) Find all inflection points of \( f \).
(g) Draw a nice graph of \( f \).