Math 131 - March 21, 2016
Solutions

Warm-up Problems

1. What is an absolute maximum of $f(x)$? (How about an absolute minimum?)

   Solution: M is an absolute maximum if $f(x) \leq M$ for all x.

2. What is a local maximum of $f(x)$? (How about a local minimum?)

   Solution: $M = f(c)$ is a local maximum if $f(x) \leq M$ for all x near $x = c$.

3. What is the extreme value theorem and how does it help us find maxima and minima?

   Solution: If f is continuous with domain $[a, b]$ then f has an absolute maximum and absolute minimum on the domain $[a, b]$.

4. What is a critical number? (Or a critical point?) What relevance do critical numbers have to finding extrema?

5. True/False

   (a) If M is an absolute maximum then M is also a local maximum. **Solution:** True

   (b) If M is an local maximum then M is also an absolute maximum. **Solution:** False

   (c) If $f(x)$ is continuous on $[a, b]$ then $f(x)$ can have only one maximum. **Solution:** False

   (d) If $f(x)$ has an absolute maximum at $x = c$ then $f'(c) = 0$. **Solution:** False

   (e) If $f(x)$ has a local maximum at $x = c$ then $f'(c) = 0$. **Solution:** False

   (f) Some functions have local extrema but no absolute extrema. **Solution:** True

   (g) Some functions have no local extrema and no absolute extrema. **Solution:** True

6. Drawing Problems

   (a) Draw an example of a function that has domain $[0, 10]$, absolute maximum of 100 at $x = 5$ and an absolute minimum of -100 at $x = 8$.

   (b) Draw an example of a function that has domain $[0, 10]$, absolute maximum of -100 at $x = 5$ and an absolute minimum of 100 at $x = 8$.

   (c) Draw an example of a function that has domain $[0, 10]$, local maximum of -100 at $x = 5$ and an local minimum of 100 at $x = 8$.

7. Let $f(x) = 3x^3 - 3x^3$ on the domain $D = [-1, 4]$.

   (a) Find all critical points (critical numbers). **Solution:** $x = 0$ and $x = 1$.

(b) Find all absolute maxima and minima of $f(x)$ on $[-1, 4]$. **Solution:** Plug in all points into f:

x	$f(x)$	
-1	7	
0	0	
1	-1	MIN
4	512	MAX

Lecture Problems

8. Let $f(x) = x^2 - x$ on $[0, 1]$.

   (a) Find the slope of the secant line connecting $(0, f(x))$ and $(1, f(1))$. **Solution:** $m = 0$

   (b) Find the x value, c, in $(0, 1)$, such that $f'(c)$ is equal to the slope of the secant line you found.

   Solution: $c = 1/2$

9. Same as Problem 8 but with domain $[-1, 6]$.

   Solution: $m = 4$ and $c = 5/2$

10. Same as Problem 8 but with domain $[2, 10]$.

    Solution: $m = 11$ and $c = 6$

11. Same as Problem 8 but with $f(x) = |x|$ and domain $[2, 10]$.

    Solution: $m = 1$ and any $c \in (2, 10)$ will work.

12. Same as Problem 8 but with $f(x) = |x|$ and domain $[-1, 1]$.

    Solution: $m = 0$ but no $c \in (2, 10)$ will work.