Warm-up Problems

1. What is an absolute maximum of \( f(x) \)? (How about an absolute minimum?)
   Solution: \( M \) is an absolute maximum if \( f(x) \leq M \) for all \( x \).

2. What is a local maximum of \( f(x) \)? (How about a local minimum?)
   Solution: \( M = f(c) \) is a local maximum if \( f(x) \leq M \) for all \( x \) near \( x = c \).

3. What is the extreme value theorem and how does it help us find maxima and minima?
   Solution: If \( f \) is continuous with domain \([a, b]\) then \( f \) has an absolute maximum and absolute minimum on the domain \([a, b]\).

4. What is a critical number? (Or a critical point?) What relevance do critical numbers have to finding extrema?

5. True/False
   (a) If \( M \) is an absolute maximum then \( M \) is also a local maximum. Solution: True
   (b) If \( M \) is a local maximum then \( M \) is also an absolute maximum. Solution: False
   (c) If \( f(x) \) is continuous on \([a, b]\) then \( f(x) \) can have only one maximum. Solution: False
   (d) If \( f(x) \) has an absolute maximum at \( x = c \) then \( f'(c) = 0 \). Solution: False
   (e) If \( f(x) \) has a local maximum at \( x = c \) then \( f'(c) = 0 \). Solution: False
   (f) Some functions have local extrema but no absolute extrema. Solution: True
   (g) Some functions have no local extrema and no absolute extrema. Solution: True

6. Drawing Problems
   (a) Draw an example of a function that has domain \([0, 10]\), absolute maximum of 100 at \( x = 5 \) and an absolute minimum of \(-100\) at \( x = 8 \).
   (b) Draw an example of a function that has domain \([0, 10]\), absolute maximum of \(-100\) at \( x = 5 \) and an absolute minimum of 100 at \( x = 8 \).
   (c) Draw an example of a function that has domain \([0, 10]\), local maximum of \(-100\) at \( x = 5 \) and an local minimum of 100 at \( x = 8 \).

7. Let \( f(x) = 3x^3 - 3x^3 \) on the domain \( D = [-1, 4] \).
   (a) Find all critical points (critical numbers). Solution: \( x = 0 \) and \( x = 1 \).
(b) Find all absolute maxima and minima of \( f(x) \) on \([-1, 4]\).  \textbf{Solution:} Plug in all points into \( f \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>512</td>
</tr>
</tbody>
</table>

\textbf{Lecture Problems}

8. Let \( f(x) = x^2 - x \) on \([0, 1]\).
   
   (a) Find the slope of the secant line connecting \((0, f(x))\) and \((1, f(1))\).  \textbf{Solution:} \( m = 0 \)
   
   (b) Find the \( x \) value, \( c \), in \((0, 1)\), such that \( f'(c) \) is equal to the slope of the secant line you found.  \textbf{Solution:} \( c = 1/2 \)

9. Same as Problem 8 but with domain \([-1, 6]\).
   \textbf{Solution:} \( m = 4 \) and \( c = 5/2 \)

10. Same as Problem 8 but with domain \([2, 10]\).
    \textbf{Solution:} \( m = 11 \) and \( c = 6 \)

11. Same as Problem 8 but with \( f(x) = |x| \) and domain \([2, 10]\).
    \textbf{Solution:} \( m = 1 \) and any \( c \in (2, 10) \) will work.

12. Same as Problem 8 but with \( f(x) = |x| \) and domain \([-1, 1]\).
    \textbf{Solution:} \( m = 0 \) but no \( c \in (2, 10) \) will work.