Math 131 - March 7, 2015

Review for Exam 2:

1. For each of the functions, find the derivative using the limit definition of derivative
   (a) $f(x) = 3x^2 - 2x$
   (b) $f(x) = \sqrt{2x-1}$
   (c) $f(x) = \frac{1}{x-5}$
2. Find the slope of the line tangent to the graph at the given x value. Be sure to do this using limiti definition as well as the formula/rule method.
   (a) $f(x) = 4x^2 - 2x$ at $x = 2$.
   (b) $f(x) = 1/(3x)$ at $x = -3$.
   (c) $f(x) = \sqrt{x+1}$ at $x = 3$.
3. $f(x) = x^2 - 2\sqrt{x} + \frac{3}{x^3}$. $f'(1) =$
4. $2xy^2 + e^y = e$. Find $\frac{dy}{dx}$ at $(0, 1)$.
5. $\dfrac{d}{dx}\left(\dfrac{1+x^3}{1-x^3}\right) =$
6. $\dfrac{d}{dx}\left(\sqrt{1+\sin^2 3x}\right) =$
7. $f(x) = x^3 + 2x + \cos 2x$. Find $f^{(4)}(0) =$.
8. $\frac{d}{dx}\left(x^{\sqrt{x}}\right) =$
9. $y = \ln(1+x^2)$. $y' =$
10. $f(x) = 2x^{500} + 5x^4$. $f'(1) =$.
11. $\frac{d}{dt}\left(\frac{e^t}{t+1}\right) =$
12. Find the slope of the tangent to the curve $y = \sec x - 2\cos x$ at $(\pi/3, 1)$.
13. Find $(2^{\sin(\pi x)})' =$
14. If $\cos(x-y) = x$ find $\frac{dy}{dx} =$
15. If $f(x) = \log_2(1+e^x)$, find $f'(0) =$.
16. Position is $s(t) = t^2 e^{2t}$. Determine when the velocity is zero.
17. Find the equation of the tangent line at the given point: $y = x \ln x$ at $x = 1$.

18. True/False

    (a) If f and g and differentiable and $f(x) > g(x)$ then $f'(x) > g'(x)$.
    (b) If acceleration of a particle is positive then the particle is speeding up.
    (c) If f is differentiable then $\frac{d}{dx}(xf(x)) = xf'(x) + f(x)$
    (d) $\frac{d}{dt}(\ln(t^2)) = \frac{2\ln t}{t}$
    (e) $\frac{d}{dx}(\tan^2 x) = \sec^4 x$.
    (f) $\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$
    (g) $\frac{d}{dx}(\ln 10) = \frac{1}{10}$

19. Find $\dfrac{d}{dt} t^{(e^t)} =$.

20. Let $x^3 - y + y^3 = 8$. Find the slope of the tangent line at the point $(2, 1)$.

21. Let $f(x) = \sin^{-1}(x^2)$. Find $f'(x) =$.

22. Find the equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point $(0, 1)$.

23. Let $x^2 - xy + y^2 = 4$

    (a) Find where this curve crosses the x-axis.
    (b) At the x-intercepts you found, find equations of tangent lines.

24. Find the equation of the tangent line to $h(x) = \ln(x^4)$ when $x = 1$.

25. Let $f(x) = \log_8(1/x)$. Find $f'(x) =$

26. If $y = x^x$, find all points where the tangent to the curve is horizontal.

27. Suppose position of a particle is $s(t) = -16t^2 + 96t$.

    (a) Find the average velocity over the interval $[1, 3]$.
    (b) Find the instantaneous velocity when $t = 1$.

28. Suppose position is given by $s(t) = -t^3 + 6t^2 + 10$. When is the particle speeding up and when is it slowing down?

29. If position is given by $s(t) = 4 + 3t^2 - t^3$ then:

    (a) When is particle moving to the right?
    (b) When does the particle change direction?
    (c) What is the total distance traveled between $t = 1$ and $t = 3$?

    Note: this problem originally read: $s(t) = 4 + 3t^2 - t^2$.

30. Identify the limit as a derivative and compute the limit using the derivative.
$\lim_{h\to 0} \dfrac{(2+h)^{10}-1024}{h}$.

31. Suppose $f(1) = -3$, $g(1) = 1$, $f'(1) = -1$ and $g'(1) = 3$. Suppose $h(x) = f(x)g(x) + 2\sqrt{x} + \frac{1}{2\sqrt{x}} + \frac{e^x}{x} + 15$. What is $h'(1) =$

32. Let $f(x) = x^3 - 9x^2 - 16x + 1$ and $g(x) = x^3 - x^2/2 + x$. Find where the tangent line to the graph of $y = f(x)$ is parallel to the tangent line to the graph of $y = g(x)$.

33. The slope of the tangent line to $f(t) = \frac{a+e^t}{e^{2t}}$ when $t = 0$ is 5. Find $a =$.

34. Find the linearization of $f(x) = x^3$ at $a = 2$ and use it to approximate $(2.1)^3$.

35. At noon a mother stands at a crossroads and tearfully waves goodbye as her daughter heads to college, driving north at 60mph. The lingers at the crossroads for one hour and then drives east at 50mph. After one more hour, at what rate is the distance between mother and daughter increasing?

36. The linear approximation of $\sqrt[3]{x}$ at $x = 8$ can be used to approximate the value of $\sqrt[3]{9}$. What is the approximation and is it too large or too small?

37. A man is walking north at 2 ft/s while a woman walks east at 2.5 ft/s. Both are walking towards a point P. At what rate is the distance between them decreasing when the man is 3 feet from P and the woman is 4 feet from P?

38. For what values of θ, $0 \le \theta \le \pi$, does the graph of $y = \cos 2\theta$ have a horizontal tangent?

39. Let $y = x^3/64 + \sqrt[3]{x}$. Find $\left.\dfrac{dy}{dx}\right|_{x=8}$.

40. The slope to the tangent line of $f(t) = \frac{a+e^t}{e^{2t}}$ where $t = 0$ is 5. What is a?

41. Suppose $F(x) = f(g(x))$ and $g(2) = 6$, $g'(2) = 3$, $f'(2) = -1$, $f(2) = 7$, $f'(6) = -4$. Find $F'(2)$.

42. If $g(4) = 2$ and $g'(4) = 3$, what is $\left(\frac{\sqrt{x}}{g(x)}\right)'(4)$?

43. At time t (in hours), the size P of a certain population of bacteria is $P = 5^{t^2+t}$. How fast is P changing when $t = 1$?

44. Suppose $f(3) = 1$ and $f'(3) = 2$. What is the estimated value, using a linear approximation, for $f(2.99)$?

45. There is one and only one line through the point $(2, 1)$ that is tangent to the graph $y = \frac{x}{x-1}$ at some point P. What is the x-coordinate of P?

46. Find an equation for the tangent line to the curve $y = x\cos x$ at $x = \pi$.

47. Find the slope of the tangent line to the curve $x^2 - 2xy + y^3 = 1$ at $(2, 1)$.

48. Let $f(x) = (\tan^{-1} x)^2$. Find $f'(1)$

49. Find the linearization for $f(x) = \sqrt{4 + 2x}$ at $x = 0$ and use it to approximate $\sqrt{4.08}$.

50. Let $f(x) = x^{e^x}$. Find $f'(1)$

51. Let $f(x) = x \cdot \ln(e^{\sqrt{x}})$. Find $f'(1)$.

52. Find the slope of the curve $x \arctan(y) + xy = \frac{\pi+4}{4}$ at $(1, 1)$

53. The length of a rectangle is increasing at a rate of 4 cm/sec and its width is decreasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10cm, what is the rate of change of area?

54. Let $y^2 = x^3 + 3x^2$. Find an equation of the tangent line at $(1, -2)$.

55. Let $y^2 = x^3 + 3x^2$. Find where the tangent line is horizontal.