Warm-up Problems

1. A baseball diamond is a square, 90 ft on a side. A player runs from first base to second base at 15 ft/sec. At what rate is the player’s distance from third base decreasing when he is half way from first to second base?

   **Solution:** $3\sqrt{5}$

Lecture Problems

2. Compute the linearization of the functions at the given points:
   
   (a) $f(x) = \sqrt{x}$ at $x = 36$. **Solution:** $L(x) = 6 + \frac{1}{12}(x - 36)$
   
   (b) $f(x) = x^2$ at $x = 3$. **Solution:** $L(x) = 9 + 6(x - 3)$
   
   (c) $f(x) = \sin x$ at $x = \pi/6$. **Solution:** $L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \pi/6)$
   
   (d) $f(x) = e^x$ at $x = 0$. **Solution:** $L(x) = x + 1$
   
   (e) $f(x) = \ln x$ at $x = 1$. **Solution:** $L(x) = x - 1$

3. Use the linearizations in the previous problem to estimate the function and compare to the actual function value. (Is the linearization approximation close?)

   (a) $f(37) \approx 6.08333$
   
   (b) $f(4) \approx 15$
   
   (c) $f(\pi/9) \approx .34885$
   
   (d) $f(0.5) \approx 1.5$
   
   (e) $f(1.1) \approx .1$

4. Find the derivative. Write the derivative both in hyperbolic form but also exponential form.

   (a) $f(x) = \sinh(x^2)$. $f'(x) = 2x \cos x^2 = x(e^{x^2} + e^{-x^2})$

   (b) $f(x) = \tanh(x) = \frac{\sinh x}{\cosh x}$. $f'(x) = 1 - \tanh^2 x = \sech^2 x = \frac{4}{(e^x + e^{-x})^2}$

   (c) $f(x) = \cosh^2(2x - 1)$. 
   
   $f'(x) = 4 \cosh(2x - 1) \sinh(2x - 1) = e^{-4x-2} (e^{2x} - e) (e^{2x} + e) (e^{4x} + e^2)$
Review Problems

5. For each of the functions, find the derivative using the limit definition of derivative

(a) \( f(x) = 3x^2 - 2x \)
(b) \( f(x) = \sqrt{2x - 1} \)
(c) \( f(x) = \frac{1}{x-5} \)

6. Find the slope of the line tangent to the graph at the given \( x \) value. Be sure to do this using limiti definition as well as the formula/rule method.

(a) \( f(x) = 4x^2 - 2x \) at \( x = 2 \).
   Solution: \( m = 14 \)
(b) \( f(x) = \frac{1}{3x} \) at \( x = -3 \).
   Solution: \( m = -1/27 \)
(c) \( f(x) = \sqrt{x+1} \) at \( x = 3 \).
   Solution: \( m = 1/4 \)

7. \( f(x) = x^2 - 2\sqrt{x} + \frac{3}{x^3} \). \( f'(1) = -8 \)

8. \( 2xy^2 + e^y = e \). Find \( \frac{dy}{dx} \) at \( (0, 1) \).
   Solution: \( y'(0, 1) = -2/e \)

9. \( \frac{d}{dx} \left( \frac{1 + x^3}{1 - x^3} \right) = \frac{6x^2}{(1 - x^3)^2} \)

10. \( \frac{d}{dx} \left( \sqrt{1 + \sin^2 3x} \right) = \frac{3 \sin 3x \cos 3x}{\sqrt{1 + \sin^2 3x}} \)

11. \( f(x) = x^3 + 2x + \cos 2x \). Find \( f^{(4)}(0) = 16 \).

12. \( \frac{d}{dx} \left( x\sqrt{x} \right) = x\sqrt{x} \left( \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} \right) \)

13. \( y = \ln(1 + x^2) \). \( y' = \frac{2x}{1+x^2} \)

14. \( f(x) = 2x^{500} + 5x^4 \). \( f'(1) = 1020 \).

15. \( \frac{d}{dt} \left( \frac{e^t}{t+1} \right) = \frac{te^t}{(t+1)^2} \) (FL2002, Exam 3)