Math 131 - March 4, 2016
Solutions

Warm-up Problems

1. A baseball diamond is a square, 90 ft on a side. A player runs from first base to second base at 15 ft/sec. At what rate is the player's distance from third base decreasing when he is half way from first to second base?

   Solution: $3\sqrt{5}$

Lecture Problems

2. Compute the linearization of the functions at the given points:

   (a) $f(x) = \sqrt{x}$ at $x = 36$. **Solution:** $L(x) = 6 + \frac{1}{12}(x - 36)$

   (b) $f(x) = x^2$ at $x = 3$. **Solution:** $L(x) = 9 + 6(x - 3)$

   (c) $f(x) = \sin x$ at $x = \pi/6$. **Solution:** $L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \pi/6)$

   (d) $f(x) = e^x$ at $x = 0$. **Solution:** $L(x) = x + 1$

   (e) $f(x) = \ln x$ at $x = 1$. **Solution:** $L(x) = x - 1$

3. Use the linearizations in the previous problem to estimate the function and compare to the actual function value. (Is the linearization approximation close?)

   (a) $f(37) \approx 6.08333$

   (b) $f(4) \approx 15$

   (c) $f(\pi/9) \approx .34885$

   (d) $f(0.5) \approx 1.5$

   (e) $f(1.1) \approx .1$

4. Find the derivative. Write the derivative both in hyperbolic form but also exponential form.

   (a) $f(x) = \sinh(x^2)$. $f'(x) = 2x \cos x^2 = x(e^{x^2} + e^{-x^2})$

   (b) $f(x) = \tanh(x) = \frac{\sinh x}{\cosh x}$. $f'(x) = 1 - \tanh^2 x = \text{sech}^2 x = \frac{4}{(e^x + e^{-x})^2}$

   (c) $f(x) = \cosh^2(2x - 1)$.
   $f'(x) = 4\cosh(2x - 1)\sinh(2x - 1) = e^{-4x-2}\ (e^{2x} - e)\ (e^{2x} + e)\ (e^{4x} + e^2)$

Review Problems

5. For each of the functions, find the derivative using the limit definition of derivative

   (a) $f(x) = 3x^2 - 2x$

   (b) $f(x) = \sqrt{2x-1}$

   (c) $f(x) = \frac{1}{x-5}$

6. Find the slope of the line tangent to the graph at the given x value. Be sure to do this using limiti definition as well as the formula/rule method.

   (a) $f(x) = 4x^2 - 2x$ at $x = 2$.
   Solution: $m = 14$

   (b) $f(x) = 1/(3x)$ at $x = -3$.
   Solution: $m = -1/27$

   (c) $f(x) = \sqrt{x+1}$ at $x = 3$.
   Solution: $m = 1/4$

7. $f(x) = x^2 - 2\sqrt{x} + \frac{3}{x^3}$. $f'(1) = -8$

8. $2xy^2 + e^y = e$. Find $\frac{dy}{dx}$ at $(0, 1)$.
   Solution: $y'(0, 1) = -2/e$

9. $\dfrac{d}{dx}\left(\dfrac{1+x^3}{1-x^3}\right) = \dfrac{6x^2}{(1-x^3)^2}$

10. $\dfrac{d}{dx}\left(\sqrt{1+\sin^2 3x}\right) = \dfrac{3\sin 3x \cos 3x}{\sqrt{1+\sin^2 3x}}$

11. $f(x) = x^3 + 2x + \cos 2x$. Find $f^{(4)}(0) = 16$.

12. $\frac{d}{dx}\left(x^{\sqrt{x}}\right) = x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$

13. $y = \ln(1 + x^2)$. $y' = \frac{2x}{1+x^2}$

14. $f(x) = 2x^{500} + 5x^4$. $f'(1) = 1020$.

15. $\frac{d}{dt}\left(\frac{e^t}{t+1}\right) = \frac{te^t}{(t+1)^2}$ (FL2002, Exam 3)