Math 131 - February 22, 2015
Solutions

Warm-up Problems

1. Find the derivatives.

   (a) $\frac{d}{dx}(x) = 1$

   (b) $\frac{d}{dx}(y) = \frac{dy}{dx} = y'$

2. The equation of the unit circle is $x^2 + y^2 = 1$. Find the equation of the line tangent to the unit circle when $x = 0$.

   Solution: There are two lines. $y = 1$ and $y = -1$.

3. Find the derivative of $y = \sqrt{1 - x^2}$.

Lecture Problems

4. Use implicit differentiation.

   (a) $10x^2 + y^2 = 25$. $y' = -\frac{10x}{y}$

   (b) $x^3 + y^3 = 4$. $y' = -\frac{x^3}{y^3}$

   (c) $3\sin(x + y) = x$. $y' = -\frac{2\cos(x+y)-1}{2\cos(x+y)}$

   (d) $x^3y^5 + 3x = 8y^3 + 1$. $y' = -\frac{3x^2y^5+3}{5x^3y^4-24y^2}$

5. Find y'' for your derivatives from Problem 4.

   (a) $y'' = -\frac{10y^2+100x^2}{y^3}$

   (b) $y'' = -\frac{3x^2y^4+3x^6}{y^7}$

   (c) $y'' = \frac{\sin(x+y)}{4\cos^3(x+y)}$

   (d) $y'' = \frac{120x^7y^{12}-288x^4y^{10}-3456xy^8+90x^5y^7-1296x^2y^5-180x^3y^2+432}{125x^9y^{11}-1800x^6y^9+8640x^3y^7-13824y^5}$

6. Maybe a little easier? Find y'.

   (a) $\sin y = x$. $y' = \frac{1}{\cos y}$

   (b) $\cos y = x$. $y' = -\frac{1}{\sin y}$

   (c) $\tan y = x$. $y' = \frac{1}{\sec^2 y}$

   (d) $\sec y = x$. $y' = \frac{1}{\sec y \tan y}$

   (e) $\csc y = x$. $y' = -\frac{1}{\csc y \cot y}$

   (f) $\cot y = x$. $y' = -\frac{1}{\csc^2 y}$

7. Find the derivatives.

   (a) $y = x\tan^{-1} x$. $y' = \tan^{-1} x = \frac{x}{x^2+1}$.

   (b) $y = 5\sin^{-1} 2x - 13\cos^{-1} x^2$. $y' = \frac{10}{\sqrt{1-4x^2}} + \frac{26x}{\sqrt{1-x^4}}$

   (c) $y = \tan^{-1} x \sin^{-1} x$. $y' = \frac{\sin^{-1} x}{x^2+1} + \frac{\tan^{-1} x}{\sqrt{1-x^2}}$