1. For the functions below:

- Identify the possible points of discontinuity
- Find the left and right hand limits of the function at these point(s).
- Find the value of the function at these points
- Find \( a \) so that the function is continuous

(a) \( f(x) = \begin{cases} 
3x + 1 & \text{if } x \leq 1 \\
ax + 1 & \text{if } x > 1 
\end{cases} \)

(b) \( f(x) = \begin{cases} 
2x^2 - x & \text{if } x \leq 3 \\
ax + 1 & \text{if } x > 3 
\end{cases} \)

2. Use the fact that \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \) to compute the limits:

(a) \( \lim_{x \to 0} \frac{\tan x}{x} = 1 \)

(b) \( \lim_{x \to 0} \frac{\sin 3x}{\sin 17x} = \frac{3}{17} \)

(c) \( \lim_{x \to 0} x \csc 21x = \frac{1}{21} \)

(d) \( \lim_{x \to 0} \frac{x}{\tan 13x} = \frac{1}{13} \)

(e) \( \lim_{x \to 0} \frac{x}{\cot 13x} = 0 \)

3. For the following functions, can you define (or redefine) the function at one point so that the function is continuous. Justify your answer.

(a) \( f(x) = \frac{x^2 + 2x}{x} \)

(b) \( f(x) = \frac{\sin x}{x} \)

(c) \( f(x) = \frac{\sin 2x}{x} \)

(d) \( f(x) = \begin{cases} 
x^2 + 2x & \text{if } x < 4 \\
3 & \text{if } x > 4 
\end{cases} \)

(e) \( f(x) = \begin{cases} 
2x + 1 & \text{if } x < 2 \\
-3x + 11 & \text{if } x > 2 
\end{cases} \)

(f) \( f(x) = \begin{cases} 
2x + 1 & \text{if } x < 2 \\
-3x + 10 & \text{if } x > 2 
\end{cases} \)
4. Solve for \( x \):
   (a) \( 2^{3x+1} = 5 \)
   \[ \text{Solution: } x = \frac{1}{3}(\ln 5/\ln 2 - 1) \]
   (b) \( e^{3x} = 8^{1+x} \)
   \[ \text{sol } x = \ln 8/(3 - \ln 8) \]
   (c) \( \frac{10}{e^{5x} + 2} = 1 \)
   \[ \text{Solution: } x = -\ln 8/5 \]

5. Suppose \( \cos x < 0 \) and \( \sin x = 4/5 \). \( \cot x = -3/4 \)

6. Suppose \( 1 + \ln(x + 1) \leq f(x) \leq 2x + e^x \). Find \( \lim_{x \to 0} f(x) = 1 \)

7. Find the limits:
   (a) \( \lim_{x \to 2} \frac{\sqrt{x^2+8}}{2x+1} = 4/5 \)
   (b) \( \lim_{x \to 1} \frac{1-\sqrt{x}}{1-x} = .5 \)
   (c) \( \lim_{x \to -2} \sqrt{\frac{x^2+3x+2}{x^2-4}} = 1/2 \)
   (d) \( \lim_{x \to 1} -\frac{1}{|x-1|} + \frac{1}{x-1} = 0 \)
   (e) \( \lim_{x \to 0} -\frac{4x+|x|}{x} = 3 \)
   (f) \( \lim_{t \to 3} \frac{3t^2-27}{t^2-t-6} = 18/5 \)
   (g) \( \lim_{x \to 1} \frac{(x^3-x)\sin(x-1)\sin(3(x-1))}{x(x-1)^3} = 6 \)
   (h) \( \lim_{t \to 0} \sqrt{t^2+9-3t} = 1/6 \)

8. For what value of \( a \) is \( \lim_{x \to a} \frac{(x+3)(x^2+4x+4)}{x-a} = 1 \)
   \[ \text{Solution: } a = -3 \]

9. Let \( f(x) = \begin{cases} \frac{\sin(k(x-1))}{2 + \frac{k^2x^2+kx-2k}{x-1}} & \text{if } x < 1 \\ \frac{\sin(k(x-1))}{x-1} & \text{if } x > 1 \end{cases} \)
   For what values of \( k \) will \( \lim_{x \to 1} f(x) \) exist?
   \[ \text{Solution: } k = -1 \]

10. Find a value of \( a \) so that the function is continuous: \( f(x) = \begin{cases} 3x + a & \text{if } x < 2 \\ ax^2 - 2x + 4 & \text{if } x \geq 2 \end{cases} \)
    \[ \text{Solution: } a = 2 \]

11. Find a value of \( b \) so that the function is continuous: \( f(x) = \begin{cases} bx + 5 & \text{if } x < 1 \\ x^2 + bx + 2b & \text{if } x \geq 1 \end{cases} \)
    \[ \text{Solution: } b = 2 \]

12. Let \( f(x) = x^3 - 5x \). Find the slope of the secant between \( x = -1 \) and \( x = 1 \).
    \[ \text{Solution: } -5 \]

13. Find the domain:
(a) \( f(x) = \sqrt{9 - (x + 1)^2} \)
Solution: \([-4, 2]\)

(b) \( f(x) = \sqrt{1 - (x + 1)^2}/9 \)
Solution: \([-4, 2]\)

(c) \( g(x) = \sqrt{18 - 2x} \)
Solution: \((–\infty, 9]\)

14. Let \( f(x) = 2/(x + 1) \), \( g(x) = \sqrt{x + 2} \) and \( h(x) = x + 3 \). Find \( g \circ h \circ f(1) = \sqrt{6} \)

15. Let \( f(x) = x^2 \), \( g(x) = \sqrt{1 + \ln x} \) and \( h(x) = e^{4x} \). Find \( f \circ g \circ h(1) \).
Solution: 5

16. Let \( f(x) = x/(x - 1) \) and \( g(x) = ax \) for some constant \( a \). Find the value \( a \) so that \( f \circ g(4) = 2 \).
Solution: \( a = 1/2 \)

17. Let \( f(x) = x^3 + 4 \). Find \( f^{-1}(12) \).

18. Let \( f(x) = x^4 + 4 \). Find \( f^{-1}(5) \).

19. Find \( \log_2 40 - \log_2 5 = 3 \)

20. Let \( f(x) = x/(x + 1) \) and \( g(x) = x + a \) for some \( a \). For what value of \( a \) does the graph of \( f \circ g \) have an \( x \)-intercept at \( x = 9 \)?
Solution: \( a = -9 \)

21. Find the inverse of the function

(a) \( g(x) = (x + 2)/(3x - 1) \).
Solution: \( g^{-1}(x) = (x + 2)/(3x - 1) \)

(b) \( f(x) = (e^x - 1)/(e^x + 1) \)
Solution: \( f^{-1}(x) = \ln((1 + x)/(1 - x)) \)

22. True/False

(a) If both \( f(x) \) and \( g(x) \) have domain \( D \), then the domain of \( f + g \) is also \( D \).
Solution: True

(b) In computing \( \lim_{x \to a} f(x) \), the value of \( f(a) \) is irrelevant.
Solution: True

(c) If \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \) then \( \lim_{x \to a} f(x)/g(x) \) will not exist.
Solution: False

(d) If \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \) then \( \lim_{x \to a} f(x)/g(x) \) will exist.
Solution: False

(e) The equation \( (\ln x)^6 = 6 \ln x \) holds for all real numbers \( x > 0 \)?
Solution: False
(f) If \( f(x) = x^3 \) then \( f(x + 2) = x^3 + 2 \)
Solution: False

(g) If \( f(x) = x + 5 \) then \( f^{-1}(x) = 1/(x + 5) \)
Solution: False

(h) \( \log_4 7 + \log_4 3 = \log_4 10 \)
Solution: False

(i) \( 2 \ln x = (\ln x)^2 \)
Solution: False

(j) \( f(x) = \sin x \) has an inverse and that inverse is \( \sin^{-1} x \) or \( \arcsin(x) \).
Solution: False

23. Let \( f(x) = \sqrt{4 - 2x} \)

(a) Domain of \( f \) is: \((-\infty, 2]\)

(b) Show that \( f \) is one to one.

(c) Find a formula for \( f^{-1}(x) = 2 - x^2/2 \)