Math 131 - February 8, 2015
Solutions

1. For the functions below:

   - Identify the possible points of discontinuity
   - Find the left and right hand limits of the function at these point(s).
   - Find the value of the function at these points
   - Find a so that the function is continuous

   (a) $f(x) = \begin{cases} 3x+1 & \text{if } x \leq 1 \\ ax+1 & \text{if } x > 1 \end{cases}$

   (b) $f(x) = \begin{cases} 2x^2 - x & \text{if } x \leq 3 \\ ax+1 & \text{if } x > 3 \end{cases}$

2. Use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to compute the limits:

   (a) $\lim_{x\to 0} \frac{\tan x}{x} = 1$

   (b) $\lim_{x\to 0} \frac{\sin 3x}{\sin 17x} = 3/17$

   (c) $\lim_{x\to 0} x \csc 21x = 1/21$

   (d) $\lim_{x\to 0} \frac{x}{\tan 13x} = 1/13$

   (e) $\lim_{x\to 0} \frac{x}{\cot 13x} = 0$

3. For the following functions, can you define (or redefine) the function at one point so that the function is continus. Justify your answer.

   (a) $f(x) = \frac{x^2+2x}{x}$

   (b) $f(x) = \frac{\sin x}{x}$

   (c) $f(x) = \frac{\sin 2x}{x}$

   (d) $f(x) = \begin{cases} x^2+2x & \text{if } x < 4 \\ 3 & \text{if } x > 4 \end{cases}$

   (e) $f(x) = \begin{cases} 2x+1 & \text{if } x < 2 \\ -3x+11 & \text{if } x > 2 \end{cases}$

   (f) $f(x) = \begin{cases} 2x+1 & \text{if } x < 2 \\ -3x+10 & \text{if } x > 2 \end{cases}$

4. Solve for x:

   (a) $2^{3x+1} = 5$
   Solution: $x = (1/3)(\ln 5/\ln 2 - 1)$

   (b) $e^{3x} = 8^{1+x}$
   sol $x = \ln 8/(3 - \ln 8)$

   (c) $\frac{10}{e^{-5x}+2} = 1$
   Solution: $x = -\ln 8/5$

5. Suppose $\cos x < 0$ and $\sin x = 4/5$. $\cot x = -3/4$

6. Suppose $1 + \ln(x+1) \leq f(x) \leq 2x + e^x$. Find $\lim_{x\to 0} f(x) = 1$

7. Find the limits:

   (a) $\lim_{x\to 2} \frac{\sqrt{x^3+8}}{2x+1} = 4/5$

   (b) $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = .5$

   (c) $\lim_{x\to -2} \sqrt{\frac{x^2+3x+2}{x^2-4}} = 1/2$

   (d) $\lim_{x\to 1^-} \frac{1}{|x-1|} + \frac{1}{x-1} = 0$

   (e) $\lim_{x\to 0^-} \frac{4x+|x|}{x} = 3$

   (f) $\lim_{t\to 3} \frac{3t^2-27}{t^2-t-6} = 18/5$

   (g) $\lim_{x\to 1} \frac{(x^3-x)\sin(x-1)\sin(3(x-1))}{x(x-1)^3} = 6$

   (h) $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} = 1/6$

8. For what value of a is $\lim_{x\to a} \frac{(x+3)(x^2+4x+4)}{x-a} = 1$
   Solution: $a = -3$

9. Let $f(x) = \begin{cases} \frac{\sin(k(x-1))}{x-1} & \text{if } x < 1 \\ 2 + \frac{kx^2+kx-2k}{x-1} & \text{if } x > 1 \end{cases}$ For what values of k will $\lim_{x\to 1} f(x)$ exist?
   Solution: $k = -1$

10. Find a value of a so that the function is continuous: $f(x) = \begin{cases} 3x + a & \text{if } x < 2 \\ ax^2 - 2x + 4 & \text{if } x \geq 2 \end{cases}$

    Solution: $a = 2$

11. Find a value of b so that the function is continuous: $f(x) = \begin{cases} bx + 5 \text{ if } x < 1 \\ x^2 + bx + 2b & \text{if } x \geq 1 \end{cases}$

    Solution: $b = 2$

12. Let $f(x) = x^2 - 5x$. Find the slope of the secant between $x = -1$ and $x = 1$.
    Solution: -5

13. Find the domain:

(a) $f(x) = \sqrt{9-(x+1)^2}$
Solution: $[-4, 2]$

(b) $f(x) = \sqrt{1-(x+1)^2/9}$
Solution: $[-4, 2]$

(c) $g(x) = \sqrt{18-2x}$
Solution: $(-\infty, 9]$

14. Let $f(x) = 2/(x+1)$, $g(x) = \sqrt{x+2}$ and $h(x) = x+3$. Find $g \circ h \circ f(1) = \sqrt{6}$

15. Let $f(x) = x^2$, $g(x) = \sqrt{1+\ln x}$ and $h(x) = e^{4x}$. Find $f \circ g \circ h(1)$.
Solution: 5

16. Let $f(x) = x/(x-1)$ and $g(x) = ax$ for some constant a. Find the value a so that $f \circ g(4) = 2$.
Solution: $a = 1/2$

17. Let $f(x) = x^3 + 4$. Find $f^{-1}(12)$.

18. Let $f(x) = x^4 + 4$. Find $f^{-1}(5)$.

19. Find $\log_2 40 - \log_2 5 = 3$

20. Let $f(x) = x/(x+1)$ and $g(x) = x + a$ for some a. For what value of a does the graph of $f \circ g$ have an x-intercept at $x = 9$?
Solution: $a = -9$

21. Find the inverse of the function

(a) $g(x) = (x+2)/(3x-1)$.
Solution: $g^{-1}(x) = (x+2)/(3x-1)$

(b) $f(x) = (e^x - 1)/(e^x + 1)$
Solution: $f^{-1}(x) = \ln((1+x)/(1-x))$

22. True/False

(a) If both $f(x)$ and $g(x)$ have domain D, then the domain of $f+g$ is also D.
Solution: True

(b) In computing $\lim_{x\to a} f(x)$, the value of $f(a)$ is irrelevant.
Solution: True

(c) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)}$ will not exist.
Solution: False

(d) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)}$ will exist.
Solution: False

(e) The equation $(\ln x)^6 = 6 \ln x$ holds for all real numbers $x > 0$?
Solution: False

(f) If $f(x) = x^3$ then $f(x+2) = x^3 + 2$
Solution: False

(g) If $f(x) = x + 5$ then $f^{-1}(x) = 1/(x+5)$
Solution: False

(h) $\log_4 7 + \log_4 3 = \log_4 10$
Solution: False

(i) $2 \ln x = (\ln x)^2$
Solution: False

(j) $f(x) = \sin x$ has an inverse and that inverse is $\sin^{-1} x$ or $\arcsin(x)$.
Solution: False

23. Let $f(x) = \sqrt{4 - 2x}$

(a) Domain of f is: $(-\infty, 2]$

(b) Show that f is one to one.

(c) Find a formula for $f^{-1}(x) = 2 - x^2/2$