

Math 131 - February 8, 2015

1. For the functions below:

- Identify the possible points of discontinuity
- Find the left and right hand limits of the function at these point(s).
- Find the value of the function at these points
- Find a so that the function is continuous

$$(a) f(x) = \begin{cases} 3x + 1 & \text{if } x \leq 1 \\ ax + 1 & \text{if } x > 1 \end{cases}$$

$$(b) f(x) = \begin{cases} 2x^2 - x & \text{if } x \leq 3 \\ ax + 1 & \text{if } x > 3 \end{cases}$$

2. Use the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ to compute the limits:

$$(a) \lim_{x \rightarrow 0} \frac{\tan x}{x} =$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 17x} =$$

$$(c) \lim_{x \rightarrow 0} x \csc 21x =$$

$$(d) \lim_{x \rightarrow 0} \frac{x}{\tan 13x} =$$

$$(e) \lim_{x \rightarrow 0} \frac{x}{\cot 13x} =$$

3. For the following functions, can you define (or redefine) the function at one point so that the function is continus. Justify your answer.

$$(a) f(x) = \frac{x^2 + 2x}{x}$$

$$(b) f(x) = \frac{\sin x}{x}$$

$$(c) f(x) = \frac{\sin 2x}{x}$$

$$(d) f(x) = \begin{cases} x^2 + 2x & \text{if } x < 4 \\ 3 & \text{if } x > 4 \end{cases}$$

$$(e) f(x) = \begin{cases} 2x + 1 & \text{if } x < 2 \\ -3x + 11 & \text{if } x > 2 \end{cases}$$

$$(f) f(x) = \begin{cases} 2x + 1 & \text{if } x < 2 \\ -3x + 10 & \text{if } x > 2 \end{cases}$$

4. Solve for x :

(a) $2^{3x+1} = 5$

(b) $e^{3x} = 8^{1+x}$

(c) $\frac{10}{e^{-5x}+2} = 1$

5. Suppose $\cos x < 0$ and $\sin x = 4/5$. $\cot x =$

6. Suppose $1 + \ln(x + 1) \leq f(x) \leq 2x + e^x$. Find $\lim_{x \rightarrow 0} f(x) =$

7. Find the limits:

(a) $\lim_{x \rightarrow 2} \frac{\sqrt{x^3+8}}{2x+1} =$

(b) $\lim_{x \rightarrow 1} \frac{1-\sqrt{x}}{1-x} =$

(c) $\lim_{x \rightarrow -2} \sqrt{\frac{x^2+3x+2}{x^2-4}} =$

(d) $\lim_{x \rightarrow 1^-} \frac{1}{|x-1|} + \frac{1}{x-1} =$

(e) $\lim_{x \rightarrow 0^-} \frac{4x+|x|}{x} =$

(f) $\lim_{t \rightarrow 3} \frac{3t^2-27}{t^2-t-6} =$

(g) $\lim_{x \rightarrow 1} \frac{(x^3-x)\sin(x-1)\sin(3(x-1))}{x(x-1)^3} =$

(h) $\lim_{t \rightarrow 0} \frac{\sqrt{t^2+9}-3}{t^2} =$

8. For what value of a is $\lim_{x \rightarrow a} \frac{(x+3)(x^2+4x+4)}{x-a} = 1$

9. Let $f(x) = \begin{cases} \frac{\sin(k(x-1))}{x-1} & \text{if } x < 1 \\ 2 + \frac{kx^2+kx-2k}{x-1} & \text{if } x > 1 \end{cases}$ For what values of k will $\lim_{x \rightarrow 1} f(x)$ exist?

10. Find a value of a so that the function is continuous: $f(x) = \begin{cases} 3x + a & \text{if } x < 2 \\ ax^2 - 2x + 4 & \text{if } x \geq 2 \end{cases}$

11. Find a value of b so that the function is continuous: $f(x) = \begin{cases} bx + 5 & \text{if } x < 1 \\ x^2 + bx + 2b & \text{if } x \geq 1 \end{cases}$

12. Let $f(x) = x^2 - 5x$. Find the slope of the secant between $x = -1$ and $x = 1$.

13. Find the domain:

(a) $f(x) = \sqrt{9 - (x + 1)^2}$

(b) $f(x) = \sqrt{1 - (x + 1)^2/9}$

(c) $g(x) = \sqrt{18 - 2x}$

14. Let $f(x) = 2/(x + 1)$, $g(x) = \sqrt{x + 2}$ and $h(x) = x + 3$. Find $g \circ h \circ f(1) =$

15. Let $f(x) = x^2$, $g(x) = \sqrt{1 + \ln x}$ and $h(x) = e^{4x}$. Find $f \circ g \circ h(1)$.
16. Let $f(x) = x/(x - 1)$ and $g(x) = ax$ for some constant a . Find the value a so that $f \circ g(4) = 2$.
17. Let $f(x) = x^3 + 4$. Find $f^{-1}(12)$.
18. Let $f(x) = x^4 + 4$. Find $f^{-1}(5)$.
19. Find $\log_2 40 - \log_2 5 =$
20. Let $f(x) = x/(x + 1)$ and $g(x) = x + a$ for some a . For what value of a does the graph of $f \circ g$ have an x -intercept at $x = 9$?
21. Find the inverse of the function
- $g(x) = (x + 2)/(3x - 1)$.
 - $f(x) = (e^x - 1)/(e^x + 1)$
22. True/False
- If both $f(x)$ and $g(x)$ have domain D , then the domain of $f + g$ is also D .
 - In computing $\lim_{x \rightarrow a} f(x)$, the value of $f(a)$ is irrelevant.
 - If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$ then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ will not exist.
 - If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$ then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ will exist.
 - The equation $(\ln x)^6 = 6 \ln x$ holds for all real numbers $x > 0$?
 - If $f(x) = x^3$ then $f(x + 2) = x^3 + 2$
 - If $f(x) = x + 5$ then $f^{-1}(x) = 1/(x + 5)$
 - $\log_4 7 + \log_4 3 = \log_4 10$
 - $2 \ln x = (\ln x)^2$
 - $f(x) = \sin x$ has an inverse and that inverse is $\sin^{-1} x$ or $\arcsin(x)$.
23. Let $f(x) = \sqrt{4 - 2x}$
- Domain of f is:
 - Show that f is one to one.
 - Find a formula for $f^{-1}(x) =$