Warm-up Problems

1. Describe our method for finding limits (as described in class).
   (a) Try plugging in.
   (b) Try using algebra.
   (c) Try some trick (Squeeze Theorem).
   (d) Make a table or graph.

2. List the limit laws learned in class and the book.

Solution:

• \( \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) \)
• \( \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) \)
• \( \lim_{x \to a} cf(x) = c \lim_{x \to a} f(x) \)
• \( \lim_{x \to a} f(x)g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) \)
• \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \)
• \( \lim_{x \to a} [f(x)]^n = \left[ \lim_{x \to a} f(x) \right]^n \)
• \( \lim_{x \to a} x = a \)
• \( \lim_{x \to a} c = c \)

Lecture Problems

3. Given the following:

\( \lim_{x \to 2} f(x) = 4 \quad \lim_{x \to 2} g(x) = 2 \quad \lim_{x \to 2} h(x) = 0 \)

Find the following:

(a) \( \lim_{x \to 2} f(x) - 3g(x) = -2 \)
(b) \( \lim_{x \to 2} \frac{(f(x))^3 + 1}{g(x) + 3h(x)} = \frac{65}{2} \)
(c) \( \lim_{x \to 2} \frac{f(x) - 2g(x)}{h(x)} = \text{UNKNOWN} \)
(d) \( \lim_{x \to 2} \frac{f(x) - g(x)}{h(x)} = \pm \infty \)
(e) \( \lim_{x \to 2} \frac{h(x)}{f(x) - g(x)} = 0 \)
(f) \( \lim_{x \to 2} \sqrt{8f(x) - g^2(x)} = \sqrt{28} \)

4. Find the following limits (infinite limits)
(a) \( \lim_{x \to 0} \frac{1}{x^2} = \infty \)
(b) \( \lim_{x \to 0} \frac{1}{-x^2} = -\infty \)
(c) \( \lim_{x \to 0} \frac{1}{x} = DNE \)
(d) \( \lim_{x \to 0^-} \frac{1}{x} = -\infty \)
(e) \( \lim_{x \to 0^+} \frac{1}{x} = \infty \)
(f) \( \lim_{x \to 4^-} \frac{1}{x^2-16} = DNE \)
(g) \( \lim_{x \to 4^+} \frac{1}{x^2-16} = \infty \)

5. Determine the points where the function graphed below is continuous and discontinuous. Be sure to be able to explain why the function is continuous or discontinuous.