This exam has:

- 18 multiple choice questions worth 4 points each.
- 2 hand graded questions worth 14 points each.

Important:

- No graphing calculators!
  Any non-graphing, non-differentiating, non-integrating scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- For the written problems:
  Show all your work for the written problems.
  Your ability to make your solution clear will be part of the grade.
  Use the back of this sheet, if necessary.

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$	$\sin(2A) = 2 \sin A \cos A$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\cos(2A) = \cos^2 A - \sin^2 A$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\tan(2A) = \dfrac{2 \tan A}{1 - \tan^2 A}$
$\sin^2(A/2) = \dfrac{1 - \cos A}{2}$	$\cos^2(A/2) = \dfrac{1 + \cos A}{2}$
$\tan(A/2) = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$	$\log_a x = \dfrac{\log_b x}{\log_b a}$
$\sin A \sin B = \dfrac{1}{2}[\cos(A - B) - \cos(A + B)]$	$\cos A \cos B = \dfrac{1}{2}[\cos(A - B) + \cos(A + B)]$
$\sin A \cos B = \dfrac{1}{2}[\sin(A + B) + \cos(A - B)]$	$\cos A \sin B = \dfrac{1}{2}[\sin(A + B) - \cos(A - B)]$
$\sin A + \sin B = 2 \sin\left(\dfrac{A + B}{2}\right) \cos\left(\dfrac{A - B}{2}\right)$	$\sin A - \sin B = 2 \cos\left(\dfrac{A + B}{2}\right) \sin\left(\dfrac{A - B}{2}\right)$
$\cos A + \cos B = 2 \cos\left(\dfrac{A + B}{2}\right) \cos\left(\dfrac{A - B}{2}\right)$	$\cos A - \cos B = -2 \sin\left(\dfrac{A + B}{2}\right) \sin\left(\dfrac{A - B}{2}\right)$
$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$	$\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1 - x^2}}$
$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$	$\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1 + x^2}$
$\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$	$\dfrac{d}{dx}(\csc^{-1} x) = -\dfrac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$

1. Let $f(x) = 4x^3 + 3x^2 - 6x + 12$.
   Let m be the minimum of f on the interval $[-2, 1]$.
   Let M be the maximum of f on the interval $[-2, 1]$.
   Find $m + M$.

   (a) 1
   (b) 4
   (c) 1/2
   (d) 17
   (e) 13
   (f) 21
   (g) 30
   (h) f does not have a maximum or minimum.

2. Let $f(x) = x^2 - 5x + 3$ on the interval $[-2, 4]$. Let A be the average slope of f on the interval. The Mean Value Theorem guarantees a point c in the interval $(-2, 4)$ satisfying some conditions that you should know.
   Find c.

   (a) -2
   (b) -1
   (c) 0
   (d) 1
   (e) 2
   (f) 3
   (g) 4
   (h) It is not possible to find a c.

3. Let $f(x) = xe^{ax}$. Find a so that $x = 5$ is a critical point of f.

   (a) $a = -5$
   (b) $a = -1$
   (c) $a = -\frac{1}{5}$
   (d) $a = 0$
   (e) $a = 1$
   (f) $a = 5$
   (g) $a = e^5$
   (h) $a = 5e^5$
   (i) It is not possible to find such an a.

4. Let

$$f(x) = 2x^3 + 3ax^2 - a^2x + 4$$

   Find a so that f has an inflection point at $x = 3$.

   (a) -6
   (b) -4
   (c) -3
   (d) -2
   (e) 0
   (f) 2
   (g) 3
   (h) 4
   (i) 6
   (j) It is not possible to find such an a.

5. Let $f(x) = |x^2 - 4|$. Find all critical points.

   (a) 0
   (b) -2
   (c) 2
   (d) $-2, 2$
   (e) $-2, 0, 2$
   (f) There are no critical points

6. Suppose you know that $f(1) = -2$, $f'(1) = 0$ and $f''(1) = -1$.
   Choose the correct statement(s).
   You may assume that all derivatives of f exist everywhere.

   (a) f has a local minimum at $x = 1$
   (b) f has a local maximum at $x = 1$
   (c) f has an absolute minimum at $x = 1$
   (d) f has an absolute maximum at $x = 1$
   (e) f has an inflection point at $x = 1$
   (f) f has neither a maximum or minimum or inflection point at $x = 1$
   (g) None of the above are correct
   (h) More than one of the above is correct

7. Let $f(x) = x^{1/3}(x+1)^{2/3}$. Find all critical points of f.

   Let N be the number of critical points.
   Let S be the sum of these critcal points

   Find $N + S$

   (a) $-\infty$
   (b) 0
   (c) $\frac{1}{3}$
   (d) 1
   (e) $\frac{4}{3}$
   (f) $\frac{5}{3}$
   (g) 2
   (h) ∞ (there are infinitely many critical points)
   (i) There are no critical points

8. A function, $f(x)$, has derivatives:

$$\begin{aligned} f'(x) =& (x+1)^2(x-1)^2 \\ f''(x) =& 4x(x-1)(x+1) \end{aligned}$$

   Let:

$$\begin{aligned} M &= \text{Number of local maximum} \\ m &= \text{Number of local minimum} \\ I &= \text{Number of inflection points} \end{aligned}$$

   Find $M + 2m + 3I$.

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4
   (f) 6
   (g) 9
   (h) 15

Use the following for Questions 9 and 10.
Suppose you know the following data about a differentiable function $f(x)$.
Also suppose that suppose all derivatives of f exist.

x	$f(x)$	$f'(x)$	$f''(x)$
2	1	0	-1
5	0	-1	0
9	-4	0	1

9. Which of the following statements are true about the function f?

   I. f must have a local maximum at $x = 2$.
   II. f must have a local minimum at $x = 2$.
   III. f must have a local maximum at $x = 9$.
   IV. f must have a local minimum at $x = 9$.

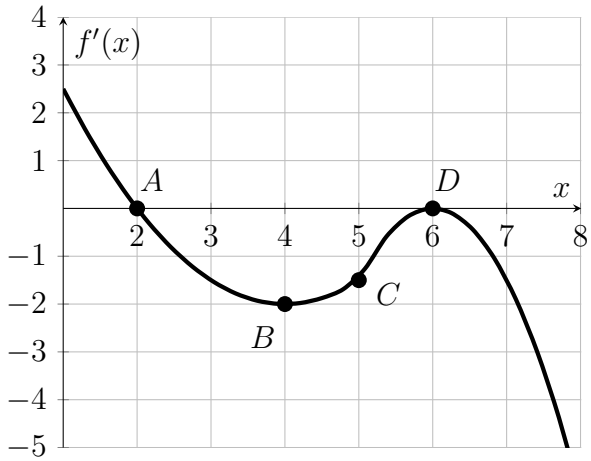
   (a) I only
   (b) II only
   (c) III only
   (d) IV only
   (e) I and III only
   (f) I and IV only
   (g) II and III only
   (h) II and IV only
   (i) None of them are true

10. Which of the following statements are true about the function f?

   I. f must have an inflection point at $x = 5$
   II. f could have an inflection point at $x = 5$
   III. f must have an inflection point somewhere in the interval $(2, 9)$

   (a) I only
   (b) II only
   (c) III only
   (d) I and II only
   (e) I and III only
   (f) II and III only
   (g) I, II and III
   (h) None of them are true

The following will be used for Questions 11 and 12.
This is a graph of $f'(x)$.
Points $A = (2, 0)$, $B = (4, -2)$, $C = (5, -3/2)$ and $D = (6, 0)$ are labeled.
Determine the x-value of all local maxima, local minima and inflection points.



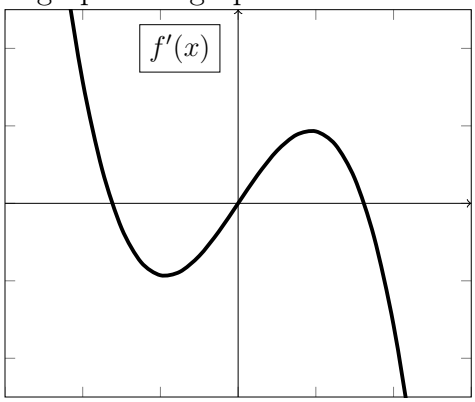
11. Where does $f(x)$ have a local maxima on the interval $[0, 8]$?

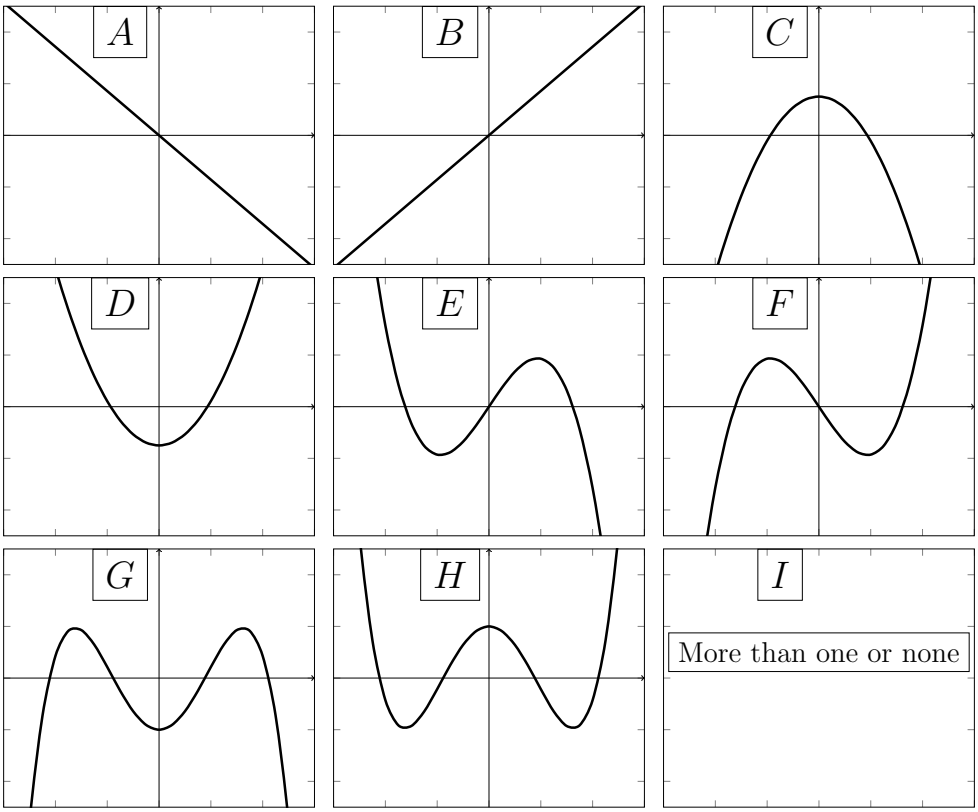
    (a) $x = 2$ (Point A)
    (b) $x = 4$ (Point B)
    (c) $x = 5$ (Point C)
    (d) $x = 6$ (Point D)
    (e) $f(x)$ has more than one local maxima
    (f) $f(x)$ does not have any local maxima
    (g) There is not enough information to determine the answer

12. Select which of the following are where $f(x)$ has an inflection point on the interval $[0, 8]$.

    (a) $x = 2$ (Point A)
    (b) $x = 4$ (Point B)
    (c) $x = 5$ (Point C)
    (d) $x = 6$ (Point D)
    (e) $f(x)$ has more than one inflection point
    (f) $f(x)$ does not have any inflection points
    (g) There is not enough information to determine the answer

Use the following graph of $f'(x)$ and the answers given below for Questions 13 and 14.
All graphs are graphed in the same graphing window (same scales for all graphs).





13. Select the graph that represents $f(x)$.

14. Select the graph that best represents $f''(x)$.

15. Find

$$\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)}$$

(a) $-\pi$
(b) -1
(c) $-1/\pi$
(d) 0
(e) 1
(f) $1/\pi$
(g) π
(h) DNE: Limit does not exist

16. Find

$$\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^{x/5}$$

(a) 0
(b) $\frac{2}{5}$
(c) 1
(d) $e^{2/5}$
(e) $\frac{5}{2}$
(f) e
(g) e^2
(h) DNE: Limit does not exist

17. Find

$$\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 + 3x - 5}$$

(a) $-5/2$
(b) -1
(c) $-2/5$
(d) 0
(e) $2/5$
(f) 1
(g) $5/2$
(h) DNE: Limit does not exist

18. Find

$$\lim_{x \to 0^+} x^{(x^2)}$$

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) 2
(e) e
(f) e^2
(g) DNE: Limit does not exist

WRITTEN PROBLEM—SHOW YOUR WORK

Note: Your discussion section letter can be found on the front cover of exam.

Name:

ID: Discussion Section:

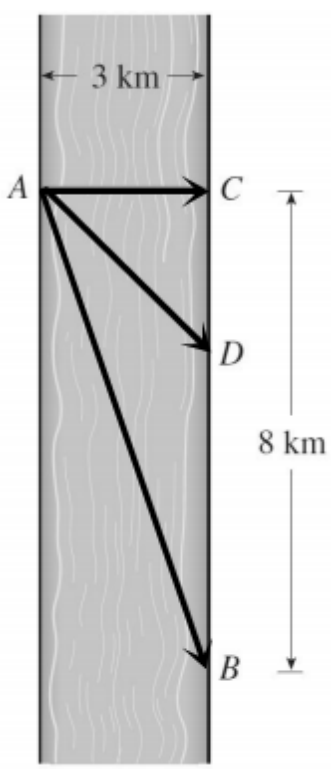
19. A man launches his boat from point A on a bank of a straight river (with no current), 3 km wide, and wants to reach point B, 8 km downstream on the oposite bank, as quickly as posible.
He could proced in one of three ways:

    I. Row his boat directly acros the river to point C and then run to B.

    II. Row directly to B.

    III. Row to some point D between C and B and then run to B.

    If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as posible? And, how much time will it take him to arrive at B?



WRITTEN PROBLEM—SHOW YOUR WORK

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Name:

ID: Discussion Section:

20. Let $f(x) = x^4 - 8x^3 + 18x^2 - 8$.

    (a) Find all critical points, maxima, minima, possible inflection points, and inflection points.

    (b) Draw the graph. Be sure to label all important points and graph features on your graph.