This exam has:

- 18 multiple choice questions worth 4 points each.
- 2 hand graded questions worth 14 points each.

Important:

- No graphing calculators!
  Any non-graphing, non-differentiating, non-integrating scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- For the written problems:
  Show all your work for the written problems.
  Your ability to make your solution clear will be part of the grade.
  Use the back of this sheet, if necessary.

\[
\begin{align*}
\sin(A \pm B) &= \sin A \cos B \pm \sin B \cos A & \sin(2A) &= 2 \sin A \cos A \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B & \cos(2A) &= \cos^2 A - \sin^2 A \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} & \tan(2A) &= \frac{2 \tan A}{1 - \tan^2 A} \\
\sin^2(A/2) &= \frac{1 - \cos A}{2} & \cos^2(A/2) &= \frac{1 + \cos A}{2} \\
\tan(A/2) &= \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A} & \log_a x = \frac{\log_b x}{\log_b a} \\
\sin A \sin B &= \frac{1}{2} [\cos(A - B) - \cos(A + B)] & \cos A \cos B &= \frac{1}{2} [\cos(A - B) + \cos(A + B)] \\
\sin A \cos B &= \frac{1}{2} [\sin(A + B) + \cos(A - B)] & \cos A \sin B &= \frac{1}{2} [\sin(A + B) - \cos(A - B)] \\
\sin A + \sin B &= 2 \sin \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right) & \sin A - \sin B &= 2 \cos \left(\frac{A + B}{2}\right) \sin \left(\frac{A - B}{2}\right) \\
\cos A + \cos B &= 2 \cos \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right) & \cos A - \cos B &= -2 \sin \left(\frac{A + B}{2}\right) \sin \left(\frac{A - B}{2}\right) \\
\frac{d}{dx} (\sin^{-1} x) &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} (\cos^{-1} x) &= -\frac{1}{\sqrt{1 - x^2}} \\
\frac{d}{dx} (\tan^{-1} x) &= \frac{1}{1 + x^2} & \frac{d}{dx} (\cot^{-1} x) &= -\frac{1}{1 + x^2} \\
\frac{d}{dx} (\sec^{-1} x) &= \frac{1}{|x|\sqrt{x^2 - 1}} & \frac{d}{dx} (\csc^{-1} x) &= -\frac{1}{|x|\sqrt{x^2 - 1}}
\end{align*}
\]
1. Let \( f(x) = 2\sqrt{x} - \frac{1}{x^3} \). Find \( f'(1) \).

   (a) \(-4\)  
   (b) \(-2\)  
   (c) \(2\)  
   (d) \(3\)  
   (e) \(\frac{7}{2}\)  
   (f) \(4\)  
   (g) \(5\)  
   (h) \(7\)

2. Find \( \frac{d}{dx} \left( \frac{x}{x + 2} \right) \)

   (a) \(1\)  
   (b) \(\frac{1}{x + 2}\)  
   (c) \(\frac{1}{(x + 2)^2}\)  
   (d) \(\frac{2}{x + 2}\)  
   (e) \(\frac{2}{(x + 2)^2}\)  
   (f) \(\frac{x}{x + 2}\)  
   (g) \(\frac{x}{(x + 2)^2}\)  
   (h) \(\frac{\pi}{2 + \pi}\)
3. Find \( \frac{d}{dt} \left( \sqrt{1 - \cos^3(t)} \right) \)

(a) \( \sqrt{1 - \cos^3(t)} \)
(b) \( \frac{1}{\sqrt{1 - \cos^3 t}} \)
(c) \( \frac{1}{2\sqrt{1 - \cos^3 t}} \)
(d) \( \frac{\cos^2 t \sin t}{\sqrt{1 - \cos^3 t}} \)
(e) \( \frac{3 \cos^2 t \sin t}{\sqrt{1 - \cos^3 t}} \)
(f) \( \frac{3 \cos^2 t \sin t}{2\sqrt{1 - \cos^3 t}} \)

4. Find \( \frac{d}{dx} \left( x^{(x^2)} \right) \)

(a) \( (x^2 - 1)x^{(x^2)} \)
(b) \( (x^2 - 1)x^{(x^2 - 1)} \)
(c) \( x^{(x^2)} \ln (x^2) \)
(d) \( x^{(x^2)}(\ln (x) + 1) \)
(e) \( x^{(x^2)}(\ln (x^2) + 1) \)
(f) \( x^{(x^2)} (2x \ln (x) + x) \)
5. Find \( \frac{d}{dx} \left( \left( \tan^{-1}(2x) \right)^2 \right) \)

(a) \( \frac{1}{1 + x^2} \)

(b) \( \frac{1}{1 + 4x^2} \)

(c) \( 3 \left( \tan^{-1}(2x) \right)^2 \)

(d) \( 3 \tan^{-2}(2x) \)

(e) \( \frac{6 \left( \tan^{-1}(2x) \right)^2}{1 + 4x^2} \)

(f) \( \frac{3 \left( \tan^{-1}(2x) \right)^2}{1 + 4x^2} \)

(g) \( \frac{\left( \tan^{-1}(2x) \right)^2}{1 + x^2} \)

6. Let \( f(t) = \ln(t^2 + 1) \). Find \( f'(1) \).

(a) 0

(b) \( \frac{1}{2} \)

(c) 1

(d) \( \frac{3}{2} \)

(e) 2

(f) \( \frac{5}{2} \)

(g) 3
7. If $y$ is a function of $x$ and

$$y^3 - 2yx = x$$

Find the slope of the tangent line at the point $(1, -1)$

(a) $-3$
(b) $-2$
(c) $-1$
(d) $0$
(e) $1$
(f) $2$
(g) $3$
(h) $4$

8. Suppose position of a particle is given by $s(t) = t^3 - t^2 - 5t$. When $t = 0$ select the true statement.

(a) The particle is speeding up
(b) The particle is slowing down
(c) The particle is at constant speed
(d) None of the above
(e) All of the above
9. If \( f(x) = \log_x 2 \) find \( f'(2) \)

(a) \(-\frac{1}{\ln 2}\)
(b) \(-1\)
(c) \(-\frac{1}{2\ln 2}\)
(d) \(-\ln 2\)
(e) \(-\frac{1}{2}\)
(f) \(\frac{1}{2}\)
(g) \(\ln 2\)
(h) \(\frac{1}{2\ln 2}\)
(i) \(1\)
(j) \(\frac{1}{\ln 2}\)

10. Suppose \( h(x) = f(x)g(x) \) and

| \( f(1) = 2 \) | \( g(1) = -1 \) |
| \( f'(1) = 3 \) | \( g'(1) = 2 \) |

Find \( h'(1) \).

(a) \(-7\)
(b) \(-5\)
(c) \(-1\)
(d) \(0\)
(e) \(1\)
(f) \(5\)
(g) \(7\)
11. A car is driven at a constant speed starting at time $t = 0$. Which of the following could be a graph of the position of the car.

![Graphs of position functions](https://i.imgur.com/5G5xG.png)

12. Suppose you have a function such that

$$f(x + h) - f(x) = h \cos h + h^2 x^2 + 3hx$$

Find $f'(1)$.

(a) $-1$
(b) $0$
(c) $1$
(d) $2$
(e) $3$
(f) $4$
(g) $5$
(h) $6$
(i) $7$
Use the graphs below for Problems 13 and 14.

13. The graphs of \( f(x) \) and \( g(x) \) are given above.
   Let \( A(x) = (f \circ g)(x) \). Find \( A'(4) \).
   
   (a) $-3$
   (b) $-2$
   (c) $-1$
   (d) $-\frac{1}{2}$
   (e) $0$
   (f) $\frac{1}{2}$
   (g) $1$
   (h) $2$
   (i) $3$

14. The graphs of \( f(x) \) and \( g(x) \) are given above.
   Let \( B(x) = (f \cdot g)(x) \). Find \( B'(4) \).
   
   (a) $-12$
   (b) $-10$
   (c) $-4$
   (d) $-2$
   (e) $-1$
   (f) $2$
   (g) $4$
   (h) $8$
Questions 15 and 16 use the following information. At \( x = 2 \), a function \( f(x) \) has tangent line

\[
y = -2x + 5
\]

15. Find \( f(2) + f'(2) \)

(a) \(-5\)  
(b) \(-4\)  
(c) \(-3\)  
(d) \(-2\)  
(e) \(-1\)  
(f) 0  
(g) 1  
(h) 2  
(i) 3  
(j) 4  
(k) 5

16. Let \( g(x) = (f(x))^3 \). Find \( g'(2) \).

(a) \(-10\)  
(b) \(-8\)  
(c) \(-6\)  
(d) \(-2\)  
(e) 0  
(f) 2  
(g) 4  
(h) 6  
(i) 8
17. Consider the curve defined by

\[ y^2 - y + 4 = x^2 \]

The point (2, 1) is on the curve. Find \( \frac{d^2 y}{dx^2} \) at the point (2, 1).

(a) -30  
(b) -25  
(c) -20  
(d) -15  
(e) -10  
(f) -5  
(g) 10  
(h) 20  
(i) 30

18. Let \( L(x) \) be the linearization of \( f(x) = x^2 + 2x \) at the point \( x = 2 \). Find \( L(1) \).

(a) -4  
(b) -2  
(c) 0  
(d) 2  
(e) 4  
(f) 6  
(g) 8  
(h) \pi/4
Note: Your discussion section letter can be found on the front cover of exam.

Name:

ID:

Discussion Section:

19. The ship “Albert” is 32 miles north of the ship “Betty” and is sailing due south at 16 mph. Betty is sailing due east at 12 mph. At what rate is the distance between the ships changing after 1 one hour?

(a) Draw a picture that shows what is going on and labels the important variables.
(b) Set up an equation relating the variables.
(c) Solve the problem.
20. The graph below is a graph of velocity of a particle, \( v(t) \).
   NOTE: it is a graph of velocity, not position, not distance.
   Let \( s(t) \) denote the position of the particle.
   
   (a) Find the slope of the tangent line on the graph of the position function, \( s(t) \), when \( t = 1 \).
   (b) For what values of \( t \) does the graph of \( s(t) \) have a horizontal tangent line?
   (c) Over the time interval \((0, 4)\), determine where the particle is speeding up and where the particle is slowing down.

Justify your answers.