This exam has:

- 18 multiple choice questions worth 4 points each.
- 2 hand graded questions worth 14 points each.

Important:

- No graphing calculators!
  Any non-graphing, non-differentiating, non-integrating scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- For the written problems:
  Show all your work for the written problems.
  Your ability to make your solution clear will be part of the grade.
  Use the back of this sheet, if necessary.

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$	$\sin(2A) = 2 \sin A \cos A$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\cos(2A) = \cos^2 A - \sin^2 A$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\tan(2A) = \dfrac{2 \tan A}{1 - \tan^2 A}$
$\sin^2(A/2) = \dfrac{1 - \cos A}{2}$	$\cos^2(A/2) = \dfrac{1 + \cos A}{2}$
$\tan(A/2) = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$	$\log_a x = \dfrac{\log_b x}{\log_b a}$
$\sin A \sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$	$\cos A \cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$
$\sin A \cos B = \dfrac{1}{2}\left[\sin(A + B) + \cos(A - B)\right]$	$\cos A \sin B = \dfrac{1}{2}\left[\sin(A + B) - \cos(A - B)\right]$
$\sin A + \sin B = 2 \sin\left(\dfrac{A + B}{2}\right) \cos\left(\dfrac{A - B}{2}\right)$	$\sin A - \sin B = 2 \cos\left(\dfrac{A + B}{2}\right) \sin\left(\dfrac{A - B}{2}\right)$
$\cos A + \cos B = 2 \cos\left(\dfrac{A + B}{2}\right) \cos\left(\dfrac{A - B}{2}\right)$	$\cos A - \cos B = -2 \sin\left(\dfrac{A + B}{2}\right) \sin\left(\dfrac{A - B}{2}\right)$
$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$	$\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1 - x^2}}$
$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$	$\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1 + x^2}$
$\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$	$\dfrac{d}{dx}(\csc^{-1} x) = -\dfrac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$

1. Let $f(x) = 2\sqrt{x} - \dfrac{1}{x^3}$. Find $f'(1)$.

   (a) -4
   (b) -2
   (c) 2
   (d) 3
   (e) $\dfrac{7}{2}$
   (f) $4 \longrightarrow$ **CORRECT**
   (g) 5
   (h) 7

   Solution: Use the Power Rule: $f'(x) = \frac{1}{\sqrt{x}} + \frac{3}{x^4}$ so $f'(1) = 4$.

2. Find $\dfrac{d}{dx}\left(\dfrac{x}{x+2}\right)$

   (a) 1
   (b) $\dfrac{1}{x+2}$
   (c) $\dfrac{1}{(x+2)^2}$
   (d) $\dfrac{2}{x+2}$
   (e) $\dfrac{2}{(x+2)^2} \longrightarrow$ **CORRECT**
   (f) $\dfrac{x}{x+2}$
   (g) $\dfrac{x}{(x+2)^2}$
   (h) $\dfrac{\pi}{2+\pi}$

   Solution: Use the quotient Rule: $f'(x) = \frac{2}{(x+2)^2}$

3. Find $\dfrac{d}{dt}\left(\sqrt{1-\cos^3(t)}\right)$

   (a) $\sqrt{1-\cos^3(t)}$

   (b) $\dfrac{1}{\sqrt{1-\cos^3 t}}$

   (c) $\dfrac{1}{2\sqrt{1-\cos^3 t}}$

   (d) $\dfrac{\cos^2 t \sin t}{\sqrt{1-\cos^3 t}}$

   (e) $\dfrac{3\ \cos^2 t \sin t}{\sqrt{1-\cos^3 t}}$

   (f) $\dfrac{3\ \cos^2 t \sin t}{2\sqrt{1-\cos^3 t}}$ $\longrightarrow$ **CORRECT**

   Solution: This is a chain rule within a chain rule.

4. Find $\dfrac{d}{dx}\left(x^{(x^2)}\right)$

   (a) $(x^2-1)x^{\left(x^2\right)}$

   (b) $(x^2-1)x^{\left(x^2-1\right)}$

   (c) $x^{(x^2)}\ln\left(x^2\right)$

   (d) $x^{(x^2)}(\ln(x)+1)$

   (e) $x^{(x^2)}\left(\ln\left(x^2\right)+1\right)$

   (f) $x^{\left(x^2\right)}\left(2\,x\ln(x)+x\right)$ $\longrightarrow$ **CORRECT**

   Solution:

$$\begin{aligned}\frac{d}{dx}\left(x^{(x^2)}\right) &= \frac{d}{dx}\left(e^{\ln x^{(x^2)}}\right) = \frac{d}{dx}\left(e^{x^2\ln x}\right)\\ &= \left(e^{x^2\ln x}\right)\left(x^2\ln x\right)' = \left(x^{(x^2)}\right)\left(2x\ln x + x^2\cdot\frac{1}{x}\right)\\ &= \left(x^{(x^2)}\right)\left(2x\ln x + x\right)'\end{aligned}$$

5. Find $\dfrac{d}{dx}\left((\tan^{-1}(2x))^3\right)$

   (a) $\dfrac{1}{1+x^2}$
   (b) $\dfrac{1}{1+4x^2}$
   (c) $3\,(\tan^{-1}(2x))^2$
   (d) $3\,\tan^{-2}(2x)$
   (e) $\dfrac{6\,(\tan^{-1}(2x))^2}{1+4x^2}$ $\longrightarrow$ **CORRECT**
   (f) $\dfrac{3\,(\tan^{-1}(2x))^2}{1+4\,x^2}$
   (g) $\dfrac{(\tan^{-1}(2x))^2}{1+x^2}$

   Solution: Use the chain rule.

$$\begin{aligned}\frac{d}{dx}\left((\tan^{-1}(2x))^3\right) =& 3\left(\tan^{-1}(2x)\right)^2\left(\tan^{-1}(2x)\right)' \\ =& 3\left(\tan^{-1}(2x)\right)^2\left(\frac{1}{1+(2x)^2}\right)(2x)' \\ =& 3\left(\tan^{-1}(2x)\right)^2\left(\frac{1}{1+(2x)^2}\right)(2)\end{aligned}$$

6. Let $f(t) = \ln(t^2+1)$. Find $f'(1)$.

   (a) 0
   (b) $\frac{1}{2}$
   (c) 1 $\longrightarrow$ **CORRECT**
   (d) $\frac{3}{2}$
   (e) 2
   (f) $\frac{5}{2}$
   (g) 3

   Solution: $f'(t) = \frac{2t}{t^2+1}$ So, $f'(1) = \frac{2}{2} = 1$

7. If y is a function of x and

$$y^3 - 2yx = x$$

Find the slope of the tangent line at the point $(1, -1)$

(a) -3
(b) -2
(c) $-1 \longrightarrow$ **CORRECT**
(d) 0
(e) 1
(f) 2
(g) 3
(h) 4

Solution: Take the derivative implicitly.

$$\begin{aligned} 3y^2\, y' - 2(y'x + y) &= 1 \\ y' &= \frac{2y+1}{3y^2 - 2x} \\ y'(1, -1) &= -1 \end{aligned}$$

8. Suppose position of a particle is given by $s(t) = t^3 - t^2 - 5t$. When $t = 0$ select the true statement.

(a) The particle is speeding up $\longrightarrow$ **CORRECT**
(b) The particle is slowing down
(c) The particle is at constant speed
(d) None of the above
(e) All of the above

Solution: Note that $s'(0) = -5$ and $s''(0) = -2$. Thus, velocity is negative and acceleration is negative. The velocity is accelerating in the direction it is traveling and thus the particle is speeding up.

9. If $f(x) = \log_x 2$ find $f'(2)$

   (a) $-\dfrac{1}{\ln 2}$
   (b) -1
   (c) $-\dfrac{1}{2\ln 2}$ $\longrightarrow$ **CORRECT**
   (d) $-\ln 2$
   (e) $-\dfrac{1}{2}$
   (f) $\dfrac{1}{2}$
   (g) $\ln 2$
   (h) $\dfrac{1}{2\ln 2}$
   (i) 1
   (j) $\dfrac{1}{\ln 2}$

   Solution: Using the change of base formula, we have $f(x) = \dfrac{\ln 2}{\ln x}$ and $f'(x) = -\dfrac{\ln 2}{x(\ln x)^2}$.

10. Suppose $h(x) = f(x)g(x)$ and

$f(1) = 2$	$g(1) = -1$
$f'(1) = 3$	$g'(1) = 2$

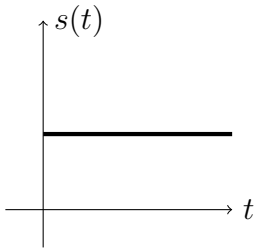
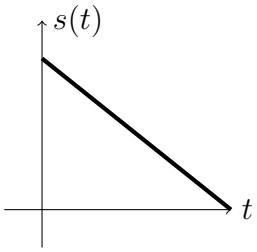
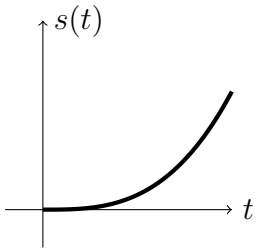
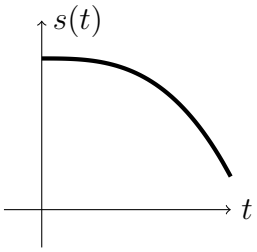
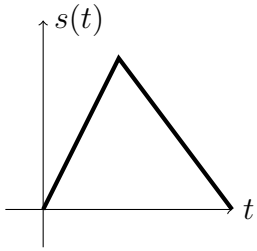
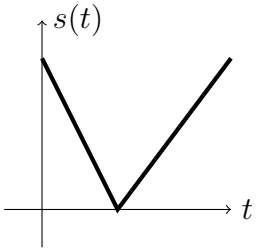
   Find $h'(1)$.

   (a) -7
   (b) -5
   (c) -1
   (d) 0
   (e) 1 $\longrightarrow$ **CORRECT**
   (f) 5
   (g) 7

   Solution:

$$\begin{aligned} h'(x) =& f'(x)g(x) + f(x)g'(x) \\ h'(1) =& f'(1)g(1) + f(1)g'(1) \\ =& 3 \cdot -1 + 2 \cdot 2 = 1 \end{aligned}$$

11. A car is driven at a constant speed starting at time $t = 0$. Which of the following could be a graph of the position of the car.

A	B	C	D
	**CORRECT** 		
E	**F**	**G**	**H**
		All could be	None could be

12. Suppose you have a function such that

$$f(x+h) - f(x) = h\cos h + h^2x^2 + 3hx$$

Find $f'(1)$.

(a) -1

(b) 0

(c) 1

(d) 2

(e) 3

(f) 4 $\longrightarrow$ **CORRECT**

(g) 5

(h) 6

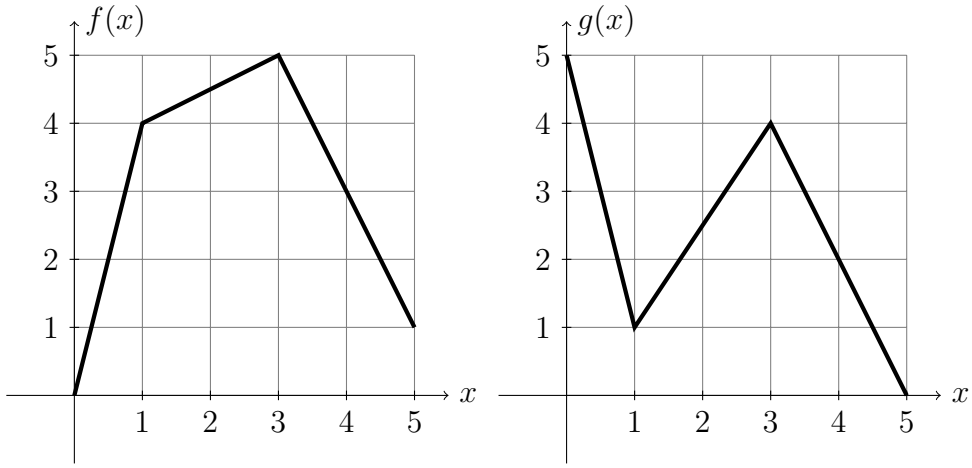
(i) 7

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{h\cos h + h^2x^2 + 3hx}{h} \\ &= \lim_{h\to 0} \cos h + hx^2 + 3x = \cos 0 + 3x = 1 + 3x \end{aligned}$$

And thus $f'(1) = 1 + 3 = 4$

Use the graphs below for Problems 13 and 14.



13. The graphs of $f(x)$ and $g(x)$ are given above.
Let $A(x) = (f \circ g)(x)$. Find $A'(4)$.

(a) -3
(b) -2
(c) -1 $\longrightarrow$ **CORRECT**
(d) $-\frac{1}{2}$
(e) 0
(f) $\frac{1}{2}$
(g) 1
(h) 2
(i) 3

Solution: Using the chain rule we have $A'(x) = (f(g(x)))' = f'(g(x))g'(x)$. And therefore

$$A'(4) = f'(g(4))g'(4) = f'(2)g'(4) = \frac{1}{2} \cdot -2 = -1$$

14. The graphs of $f(x)$ and $g(x)$ are given above.
Let $B(x) = (f \cdot g)(x)$. Find $B'(4)$.

(a) -12
(b) -10 $\longrightarrow$ **CORRECT**
(c) -4
(d) -2

(e) -1

(f) 2

(g) 4

(h) 8

Solution: Using the product rule we have $B'(x) = (f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$. And therefore

$$\begin{aligned} B'(4) = & f'(x)g(x) + f(x)g'(x) \\ & f'(4)g(4) + f(4)g'(4) \\ & (-2)(2) + (3)(-2) = -4 - 6 = -10 \end{aligned}$$

Questions 15 and 16 use the following information. At $x = 2$, a function $f(x)$ has tangent line

$$y = -2x + 5$$

15. Find $f(2) + f'(2)$

    (a) -5
    (b) -4
    (c) -3
    (d) -2
    (e) -1 $\longrightarrow$ **CORRECT**
    (f) 0
    (g) 1
    (h) 2
    (i) 3
    (j) 4
    (k) 5

    Solution: $f'(2)$ is the slope of the tangent line and is -2. Since the tangent line is equal to the function at the point $x = 2$, $f(2)$ is the value of the tangent line when $x = 2$. So, $f(2) = (-2)(2) + 5 = 1$.

    Thus, $f(2) + f'(2) = 1 - 2 = -1$

16. Let $g(x) = (f(x))^3$. Find $g'(2)$.

    (a) -10
    (b) -8
    (c) -6 $\longrightarrow$ **CORRECT**
    (d) -2
    (e) 0
    (f) 2
    (g) 4
    (h) 6
    (i) 8

    Solution: $g'(x) = 3(f(x))^2 f'(x)$ and therefore $g'(2) = 3(f(2))^2 \cdot f'(2) = 3(1)^2 \cdot (-2) = -6$

17. Consider the curve defined by

$$y^2 - y + 4 = x^2$$

The point $(2, 1)$ is on the curve.
Find $\dfrac{d^2y}{dx^2}$ at the point $(2, 1)$.

(a) $-30 \longrightarrow$ **CORRECT**
(b) -25
(c) -20
(d) -15
(e) -10
(f) -5
(g) 10
(h) 20
(i) 30

Solution: Take the derivative implicitly.

$$\begin{aligned}
2yy' - y' &= 2x \\
y' &= \frac{2x}{2y-1} \\
y'' &= \frac{2(2y-1) - (2x)(2y')}{(2y-1)^2} \\
&= \frac{4y - 2 - 4x\left(\frac{2x}{2y-1}\right)}{(2y-1)^2} \\
&= \frac{-8x^2 + 8y^2 - 8y + 2}{(2y-1)^3} \\
y''(2,1) &= -30
\end{aligned}$$

18. Let $L(x)$ be the linearization of $f(x) = x^2 + 2x$ at the point $x = 2$. Find $L(1)$.

    (a) -4
    (b) -2
    (c) 0
    (d) $2 \longrightarrow$ **CORRECT**
    (e) 4
    (f) 6
    (g) 8
    (h) $\pi/4$

    Solution: For $a = 2$ we have

$$\begin{aligned} L(x) =& f(a) + f'(a)(x-a) \\ =& 8 + 6(x-2) \\ L(1) =& 8 + 6(1-2) = 2 \end{aligned}$$

WRITTEN PROBLEM—SHOW YOUR WORK

Note: Your discussion section letter can be found on the front cover of exam.

Name:

ID: Discussion Section:

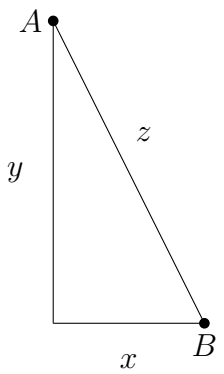
19. The ship "Albert" is 32 miles north of the ship "Betty" and is sailing due south at 16 mph. Betty is sailing due east at 12 mph. At what rate is the distance between the ships changing after 1 one hour?

    (a) Draw a picture that shows what is going on and labels the important variables.

    (b) Set up an equation relating the variables.

    (c) Solve the problem.

Solution:



We know: $\frac{dx}{dt} = 12$, $\frac{dy}{dt} = -16$. We want to find $\frac{dz}{dt}$ when $t = 1$, which means $x = 12$, $y = 16$ and $z = 20$.

The relation is $x^2 + y^2 = z^2$. Take the derivative of the relation with respect to t, solve and plug in:

$$\begin{aligned} 2x\frac{dx}{dt} + 2y\frac{dy}{dt} =& 2z\frac{dz}{dt} \\ \frac{dz}{dt} =& \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} \\ \left.\frac{dz}{dt}\right|_{t=1} =& \frac{(12)(12) + (16)(-16)}{20} = -\frac{28}{5} = -5.6\text{mph} \end{aligned}$$

WRITTEN PROBLEM—SHOW YOUR WORK

Note: Your discussion section letter can be found on the front cover of exam.

Name:

ID: Discussion Section:

20. The graph below is a graph of velocity of a particle, $v(t)$.
NOTE: it is a graph of velocity, not position, not distance.

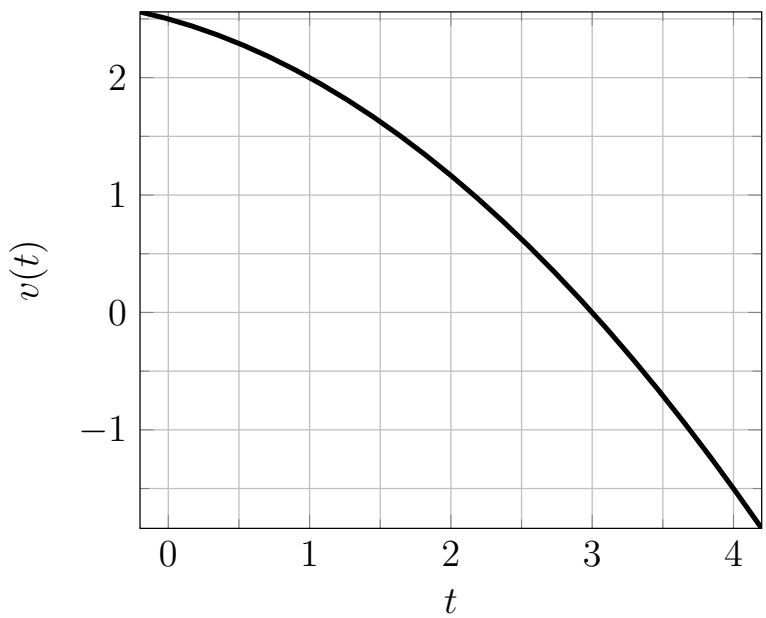
Let $s(t)$ denote the position of the particle.

(a) Find the slope of the tangent line on the graph of the position function, $s(t)$, when $t = 1$.

(b) For what values of t does the graph of $s(t)$ have a horizontal tangent line?

(c) Over the time interval $(0, 4)$, determine where the particle is speeding up and where the particle is slowing down.

Justify your answers.



Solution:

(a) The slope of the tangent line to $s(t)$ at $t = 1$ is $v(1) = 2$.

(b) $s(t)$ has a horizontal tangent when $v(t) = 0$, which is at $t = 3$

(c) Slowing down on $(0, 3)$ and speeding up on $(3, 4)$.