Lecture Problems

1. Use the Fundamental Theorem of Calculus:

\[ \int_{a}^{b} F'(x) \, dx = F(b) - F(a) \]

Steps:

- Find an antiderivative.
- Plug in the end points to the antiderivative and subtract.

(a) \( \int_{0}^{\pi} \sin(x) \, dx = \)

(b) \( \int_{1}^{5} x^2 - 2x \, dx = \)

(c) \( \int_{0}^{1} \frac{1}{x^2 + 1} \, dx = \)

(d) \( \int_{1}^{e} \frac{1}{x} \, dx = \)

2. Use the FTC to find expressions for the following functions:

(a) \( F(x) = \int_{1}^{x} t^2 \, dt = \)

(b) \( F(x) = \int_{-12}^{x} t^2 \, dt = \)

(c) \( F(x) = \int_{0}^{x} \sin(t) \, dt = \)

(d) \( F(x) = \int_{1/4 \pi}^{x} \sin(t) \, dt = \)

(e) \( F(x) = \int_{0}^{x} 5x^7 \, dt = \)

(f) \( F(x) = \int_{0}^{x} 5t^7 \, dt = \)

(g) \( F(x) = \int_{3}^{x} 5t^7 \, dt = \)

(h) \( F(x) = \int_{1}^{x} \frac{1}{t} \, dt = \)

(i) \( F(x) = \int_{5}^{x} \frac{1}{t} \, dt = \)

3. Take the derivative of the functions in Problem 2.
   Draw some conclusions for the patterns you see.

4. Use FTC (this part: \( \frac{d}{dx} \int_{a}^{x} \)), to compute the derivatives
(a) \( \frac{d}{dx} \int_{1}^{x} \sin(t^3 + 1) \, dt = \)

(b) \( \frac{d}{dx} \int_{0}^{x} \cos \left( \frac{1}{t} \right) \, dt = \)

(c) \( \frac{d}{dx} \int_{0}^{x} (-17 t^4 + t^3 + 24) \, dt = \)

(d) \( \frac{d}{dx} \int_{x}^{1} \cos \left( \frac{1}{t} \right) \, dt = \)

(e) \( \frac{d}{dx} \int_{0}^{-2x^3} \ (-17 t^4 + t^3 + 24) \, dt = \)

(f) \( \frac{d}{dx} \int_{x}^{-2x^3} \ (-17 t^4 + t^3 + 24) \, dt = \)