Warm-up Problems

1. Determine the area of the largest rectangle that can be inscribed in a circle of radius 4.

**Solution:** Draw a picture. Let \((x, y)\) be the point on the circle. Maximize \(A = 4xy\) subject to \(x^2 + y^2 = 16\). Eliminate the variable and substitute into \(A\) to get \(A = 4x\sqrt{16 - x^2}\) with domain \([0, 4]\). Critical points are \(-2\sqrt{2}, 2\sqrt{2}\).

Another approach is to maximize \(B = 16x^2y^2\) subject to same constraint. This makes derivatives and solving easier.

2. Determine the cylinder with the largest volume that can be inscribed in a cone of height 8 and base radius 4 cm.

**Solution:** Let \(x\) be the radius of the cylinder and \(y\) the height. Maximize \(V = \pi x^2y\) with constraint \(y = 8 - 2x\) (draw the picture of the cone and determine the relation between \(x\) and \(y\)). Thus, the equation become \(V = \pi x^2(8 - 2x)\) with \(x \in [0, 4]\). Critical points are \(x = 0, 8/3\). \(x = 8/3\) gives the maximum.

3. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola \(y = 9 - x^2\).

**Solution:** Draw the picture. You want to maximize \(A = 2xy\) subject to \(y = 9 - x^2\). Substituting gives \(A = 2x(9 - x^2)\) with \(x \in [0, 3]\). Critical points are \(x = \pm \sqrt{3}\).

Lecture Problems

4. Find the function \(f(x)\) that satisfies the given information.

   (a) \(f'(x) = -25\sin(5x), f(0) = -3\).
      **Solution:** \(f(x) = 5\cos 5x - 8\)

   (b) \(f''(x) = -25\sin(5x), f'(0) = -4\) and \(f(0) = -3\).
      **Solution:** \(f(x) = \sin 5x - 9x - 3\)

   (c) \(f'(x) = x^3 - 3x, f(0) = 1\)
      **Solution:** \(f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 1\)

   (d) \(f'(x) = x^3 - 3x, f(1) = 13\)
      **Solution:** \(f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + \frac{57}{4}\)

   (e) \(f'(x) = 6x^2 + 8x - 1, f(0) = 6\)
      **Solution:** \(f(x) = 2x^3 + 4x^2 - x + 6\)

   (f) \(f'(x) = 6x^2 + 8x - 1, f(0) = 6\) and \(f(1) = 11\)
      **Solution:** \(f(x) = 2x^3 + 4x^2 - x + 6\)

   (g) \(f'(x) = 6x^2 + 8x - 1, f(0) = 6\) and \(f(1) = 12\)
      **Solution:** Not possible