Math 131 - November 14, 2014
Solutions

Warm-up Problems

1. Determine the area of the largest rectangle that can be inscribed in a circle of radius 4.

   Solution: Draw a picture. Let (x, y) be the point on the circle. Maximize $A = 4xy$ subject to $x^2 + y^2 = 16$. Eliminate the variable and substitute into A to get $A = 4x\sqrt{16 - x^2}$ with domain $[0, 4]$. Critical points are $-2\sqrt{2}, 2\sqrt{2}$.

   Another approach is to maximaize $B = 16x^2y^2$ subject to same constraint. This makes derivatives and solving easier.

2. Determine the cylinder with the largest volume that can be inscribed in a cone of height 8 and base radius 4 cm .

   Solution: Let x be the radius of the cylinder and y the height. Maximize $V = \pi x^2 y$ with constraint $y = -2x + 8$ (draw the picture of the cone and determine the relation between x and y). Thus, the equation become $V = pix^2(8-2x)$ with $x \in [0, 4]$. Critical points are $x = 0, 8/3$. $x = 8/3$ gives the maximum.

3. Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 9x^2$.

   Solution: Draw the picture. You want to maximize $A = 2xy$ subject to $y = 9 - x^2$. Substituting gives $A = 2x(9 - x^2)$ with $x \in [0, 3]$. Critical points are $x = \pm\sqrt{3}$.

Lecture Problems

4. Find the function $f(x)$ that satisfies the given information.

   (a) $f'(x) = -25\sin(5x)$, $f(0) = -3$.
   Solution: $f(x) = 5\cos 5x - 8$

   (b) $f''(x) = -25\sin(5x)$, $f'(0) = -4$ and $f(0) = -3$.
   Solution: $f(x) = \sin 5x - 9x - 3$

   (c) $f'(x) = x^3 - 3x$, $f(0) = 1$
   Solution: $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 1$

   (d) $f'(x) = x^3 - 3x$, $f(1) = 13$
   Solution: $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + \frac{57}{4}$

   (e) $f'(x) = 6x^2 + 8x - 1$, $f(0) = 6$.
   Solution: $f(x) = 2x^3 + 4x^2 - x + 6$

   (f) $f'(x) = 6x^2 + 8x - 1$, $f(0) = 6$ and $f(1) = 11$
   Solution: $f(x) = 2x^3 + 4x^2 - x + 6$

   (g) $f'(x) = 6x^2 + 8x - 1$, $f(0) = 6$ and $f(1) = 12$
   Solution: Not possible