Math 131 - November 9, 2014
Solutions

Warm-up Problems

1. Write down all the indeterminate forms you can think of.
   Solution: $\frac{0}{0}$, $\infty$, $0 \cdot \infty$, $\infty - \infty$, $0^0$, $1^\infty$, $\infty^0$

2. For each of your forms in Question 1, give an example of an indeterminate form of that type.

3. Show that the indeterminant form $\frac{0}{0}$ can equal anything from $-\infty$ to $\infty$ (and everything in between).

Lecture Problems

4. Identify the indeterminate form (if any) and find the limits
   (a) $\lim_{x \to \infty} \frac{x + \sin x}{x + \cos x} = 1$
   (b) $\lim_{x \to \infty} x - \ln x = \infty$
   (c) $\lim_{x \to \infty} \frac{x}{\ln x} = \infty$
   (d) $\lim_{x \to 1^+} \frac{1}{\ln x} - \frac{1}{x - 1} = \frac{1}{2}$
   (e) $\lim_{x \to 0^+} \sin x \ln x = 0$
   (f) $\lim_{x \to \infty} x^{1/x} = 1$
   (g) $\lim_{x \to 0^+} x^x = 1$
   (h) $\lim_{x \to 0} (\tan 5x)^x = 1$
   (i) $\lim_{x \to 0} x = 1$
   (j) $\lim_{x \to \infty} \left(\frac{x + 1}{x + 2}\right)^x = e^{-1}$
   (k) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$
   (l) $\lim_{x \to 0} \frac{1}{\sin x} - \frac{1}{x} = 0$
   (m) $\lim_{x \to \infty} x - \sqrt{x^2 + 20} = 0$
   (n) $\lim_{x \to \infty} \sqrt{x + 1} - \sqrt{x} = 0$
   (o) $\lim_{x \to \infty} (x + 1)^3 - x^3 = \infty$
   (p) $\lim_{x \to \infty} x \sin\left(\frac{2}{x}\right) = 2$
\[ (q) \lim_{x \to 1} \frac{\ln x}{x^2 + x - 2} = 1/3 \]

5. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

**Solution:** \( A = xy \) with constraint \( 2x + y = 2400 \). Eliminating \( y \) gives \( A = 2400 - 2x^2 \) on the interval \([0, 1200]\). Max at \( x = 600, \ y = 1200 \).

6. Construct a window in the shape of a semi-circle over a rectangle. The distance around the outside of the window should be 12 feet. Find the dimension so that the area is as large as possible.

**Solution:** \( A = xy \) and perimeter is \( 12 = 2y + x + \pi x/2 \). Eliminate \( y \) and optimize.

7. A sheet of cardboard is 3 feet by 4 feet. You make this into a box by cutting equal size squares from each corner and folding up the edges. What is the dimension of the box with largest area?

**Solution:** \( V = (4 - 2x)(3 - 2x)(x) \).