Math 131 - October 13, 2014

Solutions

Warm-up Problems

1. Derivative Machine!
   I want you to think of yourself as a derivative computer.

   Here is a list of the derivative tools we have learned for taking derivatives. Write down what each of these means and make sure you know when to use.

   (a) Sum Rule
   (b) Constant Multiple Rule
   (c) Power Rule
   (d) Product Rule
   (e) Quotient Rule
   (f) Chain Rule
   (g) Implicit Derivatives
   (h) Exponential and Log Functions
   (i) Inverse Functions
   (j) Inverse Trig Functions

2. Write down the steps to solving a related rates problem.
   (These are the types of problems you can find in the lecture problems.)

   Solution:

   (a) Draw a picture and name variables.
   Be sure to identify which things are changing and which are constant.
   If it is changing, it get a variable.
   If it is constant, it is a constant.

   (b) What is the numerical information in terms of your variables.
   (In particular, the rates that are given and the "when ____________.")

   (c) What are you trying to find (usually a rate).

   (d) What is an equation that relates the variables you have been given.
   (In particular, your equation should relate the variables in the rate you want to find with the variable of the rate you are given.)

   (e) Do: $\dfrac{d}{dt}$.

   (f) Solve for the rate you want to find and plug in everywhere needed.

Lecture Problems

3. A ladder 20 feet long leans against a building. If the bottom of the ladder slides away from the building horizontally at a rate of 4 ft/sec, how fast is the ladder sliding down the house when the top of the ladder is 8 feet from the ground.

   Solution: Use $x^2 + y^2 = 20^2$. Take the derivative, plug in and get $\frac{dy}{dt} = -\frac{\sqrt{336}}{2}$.

4. Sand is dumped off a conveyor belt into a pile at the rate of 2 cubic feet per minute. The sand pile is shaped like a cone whose height and base diameter are always equal. At what rate is the height of the pile growing when the pile is 5 feet high?

   Solution: Key points. $V = \frac{\pi}{3}r^2 h$. Use $r = h/2$ to get $V = \frac{\pi}{12}h^3$. Take derivative and plug in to get $\frac{dh}{dt} = \frac{8}{25\pi}$ ft/min.

5. A camera is located 50 ft from a straight road along which a car is traveling at 100 ft/sec. The camera turns so that it is pointed at the car at all times. In radians per second, how fast is the camera turning as the car passes closest to the camera?

   Solution: Use $\tan\theta = x/50$ to get $d\theta/dt = -2$.

6. A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at a constant rate of 2 inches per second, how fast is the angle formed by the ladder and the ground changing (in radians per second) at the instant of the top of the ladder is 12 feet above the ground?

   Solution: Use $\cos\theta = \frac{x}{13}$. Get $\frac{d\theta}{dt} = -1/72$ radians/second.

7. A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at a constant rate of 8 inches per second, how fast is the area of the triangle formed by the ladder, building and ground changing (in square feet per second) at the instant of the top of the ladder is 12 feet above the ground?

   Solution: Use $A = \frac{1}{2}xy$. Take d/dt to get: $\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$. We are given $\frac{dx}{dt} = 8$ but we need $\frac{dy}{dt}$. To find this use $x^2 + y^2 = 13$ and take d/dt. You should get $\frac{dy}{dt}\Big|_{y=12} = -\frac{5}{18}$. Now plug all this in and get $\frac{dA}{dt} = \frac{119}{36}$ square feet per second.

8. A student is using a straw to drink from a conical paper cup with a vertical axis. She drinks at a rate of 3 cm^3/sec. If the height of the cup is 10cm and the diameter of its opening is 6cm, how fast is the level of the liquid in the cup falling when the depth of the liquid is 5cm?

   Solution: Here are the key points (without a picture): $\frac{dV}{dt} = 3$, $V = \frac{1}{3}\pi r^2 h$. Use similar triangles to find a relation between r and h and find that $r = \frac{3}{10}h$. Using this gives $V = \frac{3}{100}\pi h^3$. Working it all out eventually gives $\frac{dh}{dt} = \frac{4}{3\pi}$.