Test Review Problems

1. Solve for $x$:
   
   (a) $2^{3x+1} = 5$
   (b) $e^{3x} = 8^{1+x}$
   (c) $\frac{10}{e^{3x/2}} = 1$

2. Suppose $\cos x < 0$ and $\sin x = 4/5$. $\cot x =$

3. Suppose $1 + \ln(x + 1) \leq f(x) \leq 2x + e^x$. Find $\lim_{x \to 0} f(x) =$

4. Use your calculator to obtain a numerical estimate for the slope of the tangent line to the graph $f(x) = \sqrt{1 + \sin x}$ at $x = 0$.

5. Find the limits:
   
   (a) $\lim_{x \to 2} \frac{\sqrt{x^2+4x}}{2x+1} =$
   (b) $\lim_{x \to 1} \frac{1-\sqrt{2}}{1-x} =$
   (c) $\lim_{x \to -2} \sqrt{\frac{x^2+3x+2}{x^2-4}} =$
   (d) $\lim_{x \to 1^-} \frac{1}{|x-1|} + \frac{1}{x-1} =$
   (e) $\lim_{x \to 0^-} \frac{4x+|x|}{x} =$
   (f) $\lim_{t \to 3} \frac{3t^2-27}{t^2-9} =$
   (g) $\lim_{x \to 1} \frac{(x^3-x)\sin(x-1)\sin(3(x-1))}{x(x-1)^3} =$
   (h) $\lim_{t \to 0} \sqrt{t^2+5-3} =$

6. For what value of $a$ is $\lim_{x \to a} \frac{(x+3)(x^2+4x+4)}{x-a} = 1$

7. Let $f(x) = \begin{cases} \frac{\sin(k(x-1))}{x-1} & \text{if } x < 1 \\ \frac{2 + \frac{kx^2+kx-2k}{x-1}}{2} & \text{if } x > 1 \end{cases}$ For what values of $k$ will $\lim_{x \to 1} f(x)$ exist?

8. Find a value of $a$ so that the function is continuous: $f(x) = \begin{cases} 3x + a & \text{if } x < 2 \\ ax^2 - 2x + 4 & \text{if } x \geq 2 \end{cases}$

9. Find a value of $b$ so that the function is continuous: $f(x) = \begin{cases} bx + 5 & \text{if } x < 1 \\ x^2 + bx + 2b & \text{if } x \geq 1 \end{cases}$

10. Find the slope of the line tangent to the graph at the given $x$ value. Be sure to do this as a limit of slopes of secant lines.
    
    (a) $f(x) = 4x^2 - 2x$ at $x = 2$.
    (b) $f(x) = 1/(3x)$ at $x = -3$.
    (c) $f(x) = \sqrt{x+1}$ at $x = 3$. 
11. Let \( f(x) = x^2 - 5x \). Find the slope of the secant between \( x = -1 \) and \( x = 1 \).

12. Find the domain:
   (a) \( f(x) = \sqrt{9 - (x + 1)^2} \)
   (b) \( f(x) = \sqrt{1 - (x + 1)^2}/9 \)
   (c) \( g(x) = \sqrt{18 - 2x} \)

13. Let \( f(x) = 2/(x + 1), g(x) = \sqrt{x + 2} \) and \( h(x) = x + 3 \). Find \( g \circ h \circ f(1) \).

14. Let \( f(x) = x^2, g(x) = \sqrt{1 + \ln x} \) and \( h(x) = e^{4x} \). Find \( f \circ g \circ h(1) \).

15. Let \( f(x) = x/(x - 1) \) and \( g(x) = ax \) for some constant \( a \). Find the value of \( a \) so that \( f \circ g \circ 4 = 2 \).

16. Let \( f(x) = x^3 + 4 \). Find \( f^{-1}(12) \).

17. Let \( f(x) = x^4 + 4 \). Find \( f^{-1}(5) \).

18. Find \( \log_2 40 - \log_2 5 = \)

19. Let \( f(x) = x/(x + 1) \) and \( g(x) = x + a \) for some \( a \). For what value of \( a \) does the graph of \( f \circ g \) have an \( x \)-intercept at \( x = 9 \)?

20. Find the inverse of the function
   (a) \( g(x) = (x + 2)/(3x - 1) \).
   (b) \( f(x) = (e^x - 1)/(e^x + 1) \)

21. True/False
   (a) If both \( f(x) \) and \( g(x) \) have domain \( D \), then the domain of \( f + g \) is also \( D \).
   (b) In computing \( \lim_{x \to a} f(x) \), the value of \( f(a) \) is irrelevant.
   (c) If \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \) then \( \lim_{x \to a} \frac{f(x)}{g(x)} \) will not exist.
   (d) If \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \) then \( \lim_{x \to a} \frac{f(x)}{g(x)} \) will exist.
   (e) The equation \( (\ln x)^6 = 6 \ln x \) holds for all real numbers \( x > 0 \).
   (f) If \( f(x) = x^3 \) then \( f(x + 2) = x^3 + 2 \).
   (g) If \( f(x) = x + 5 \) then \( f^{-1}(x) = 1/(x + 5) \).
   (h) \( \log_4 7 + \log_4 3 = \log_4 10 \).
   (i) \( 2 \ln x = (\ln x)^2 \).
   (j) \( f(x) = \sin x \) has an inverse and that inverse is \( \sin^{-1} x \) or \( \arcsin(x) \).

22. Let \( f(x) = \sqrt{4 - 2x} \)
   (a) Domain of \( f \) is:
   (b) Show that \( f \) is one to one.
   (c) Find a formula for \( f^{-1}(x) = \)