Warm-up Problems

1. Use the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ to compute the limits:

(a) $\lim_{x \to 0} \frac{\sin 2x}{x} = 2$

(b) $\lim_{x \to 0} \frac{\sin x}{4x} = \frac{1}{4}$

(c) $\lim_{x \to 0} \frac{x}{\sin 4x} = \frac{1}{4}$

(d) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(e) $\lim_{x \to 0} \frac{\sin 3x}{\sin 17x} = 3/17$

(f) $\lim_{x \to 0} x \csc 21x = 1/21$

(g) More difficult, use a half angle formula: $\cos x = 1 - 2\sin^2(x/2)$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = \lim_{x \to 0} -2\frac{\sin^2(x/2)}{x} = \left(\lim_{x \to 0} -\sin(x/2)\right) \left(\lim_{x \to 0} \frac{\sin(x/2)}{x/2}\right) = 0 \cdot 1 = 0$$

2. Describe things you like and don’t like about the graph of the function below. (What is “wrong” with this function?)

I want you to articulate your issues with this function–Is it a function? Are there some $x$-values you like and some you don’t like? Why? What is the domain of the function?

![Graph of F(x)](image)

Lecture Problems

3. For the following graphs, determine the “good points” and “problem points”.

- Explain why they are points of continuity or points of discontinuity
4. Let
\[
f(x) = \begin{cases} 
  x^2 + 2x & \text{if } x \neq 4 \\
  0 & \text{if } x = 4 
\end{cases}
\]

(a) \( f(4) = 4 \)

(b) \( \lim_{x \to 4^-} f(x) = 24 \)

(c) \( \lim_{x \to 4^+} f(x) = 24 \)

(d) If \( f \) continuous?

(e) Can you change \( f \) slightly so that \( f \) is continuous?
   (Can you redefine \( f \) at one point to do this?)

5. Let
\[
f(x) = \frac{x^2 + 2x}{x + 2}
\]

(a) What is the domain of \( f \)?

(b) \( f(-2) = \text{DNE} \)

(c) \( \lim_{x \to -2^-} f(x) = -2 \)

(d) \( \lim_{x \to -2^+} f(x) = -2 \)

(e) If \( f \) continuous?

(f) Can you change \( f \) slightly so that \( f \) is continuous?
   (Can you redefine \( f \) at one point to do this?)
   \textbf{Solution:} If you make \( f(-2) = -2 \) then \( f \) will be continuous.

6. Let
\[
f(x) = \begin{cases} 
  3x + 1 & \text{if } x < -1 \\
  -2x - 4 & \text{if } x > -1 
\end{cases}
\]

(a) What is the domain of \( f \)?

(b) \( f(-1) = \text{DNE} \)

(c) \( \lim_{x \to -1^-} f(x) = -2 \)

(d) \( \lim_{x \to -1^+} f(x) = -2 \)

(e) If \( f \) continuous?

(f) Can you change \( f \) slightly so that \( f \) is continuous?
   (Can you redefine \( f \) at one point to do this?)
   \textbf{Solution:} If you make \( f(-1) = -2 \) then \( f \) will be continuous.
7. Let
\[ f(x) = \begin{cases} 
3x + 1 & \text{if } x < -1 \\
-2x + 4 & \text{if } x > -1 
\end{cases} \]

(a) What is the domain of \( f \)?
(b) \( f(-1) = \text{DNE} \)
(c) \( \lim_{x \to -1^-} f(x) = -2 \)
(d) \( \lim_{x \to -1^+} f(x) = 6 \)
(e) If \( f \) continuous?
(f) Can you change \( f \) slightly so that \( f \) is continuous?
   (Can you redefine \( f \) at one point to do this?)
   **Solution:** No

8. Let
\[ f(x) = \begin{cases} 
3x + 1 & \text{if } x \leq 1 \\
ax + 1 & \text{if } x > 1 
\end{cases} \]

(a) What is the domain of \( f \)?
(b) \( f(1) = 4 \)
(c) \( \lim_{x \to 1^-} f(x) = 4 \)
(d) \( \lim_{x \to 1^+} f(x) = a + 1 \)
(e) If \( f \) continuous?
(f) Can you find \( a \) so that \( f \) is continuous?
   **Solution:** Solve \( 4 = a + 1 \), which gives \( a = 3 \)