Math 131 - September 12, 2014
Solutions

Warm-up Problems

1. Use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to compute the limits:

   (a) $\lim_{x\to 0} \frac{\sin 2x}{x} = 2$

   (b) $\lim_{x\to 0} \frac{\sin x}{4x} = \frac{1}{4}$

   (c) $\lim_{x\to 0} \frac{x}{\sin 4x} = \frac{1}{4}$

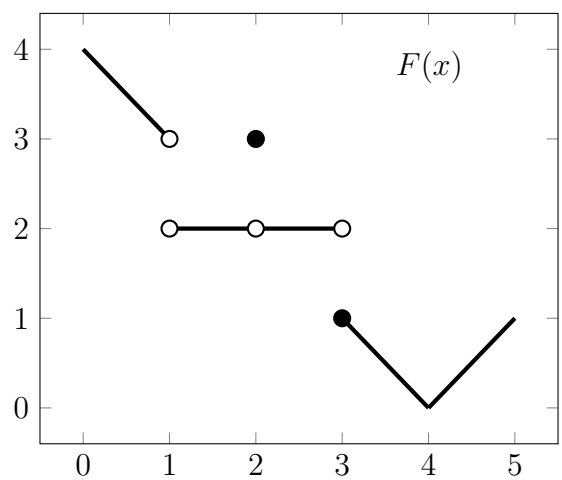
   (d) $\lim_{x\to 0} \frac{\tan x}{x} = 1$

   (e) $\lim_{x\to 0} \frac{\sin 3x}{\sin 17x} = 3/17$

   (f) $\lim_{x\to 0} x \csc 21x = 1/21$

   (g) More difficult, use a half angle formula: $\cos x = 1 - 2\sin^2(x/2)$
   $\lim_{x\to 0} \frac{\cos x - 1}{x} = \lim_{x\to 0} \frac{-2\sin^2(x/2)}{x} = \left(\lim_{x\to 0} -\sin(x/2)\right)\left(\lim_{x\to 0} \frac{\sin(x/2)}{x/2}\right) = 0 \cdot 1 = 0$

2. Describe things you like and don't like about the graph of the function below. (What is "wrong" with this function?)
   I want you to articulate your issues with this function–Is it a function? Are there some x-values you like and some you don't like? Why? What is the domain of the function?

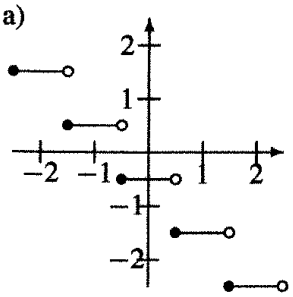


Lecture Problems

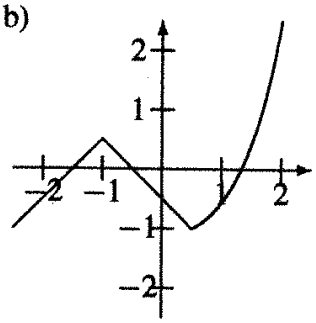
3. For the following graphs, determine the "good points" and "problem points".

   - Explain why they are points of continuity or points of discontinuity

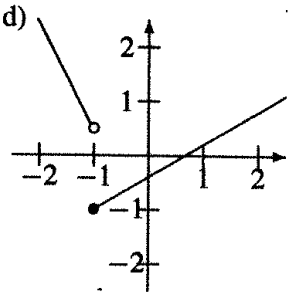
a)



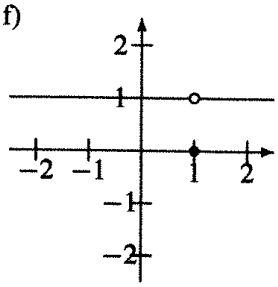
b)



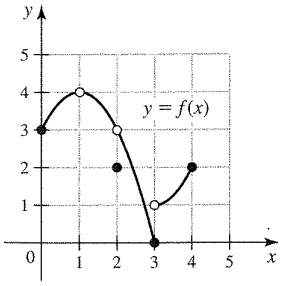
d)



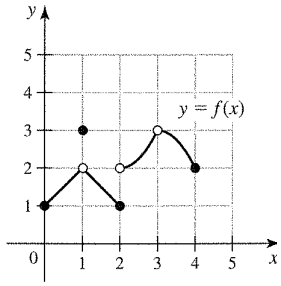
f)



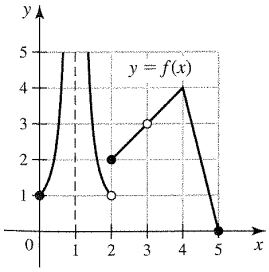
9.

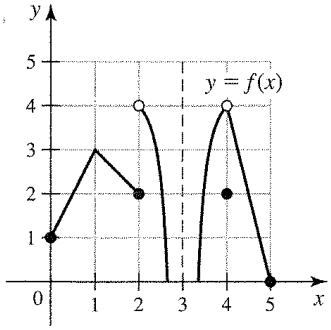


10.

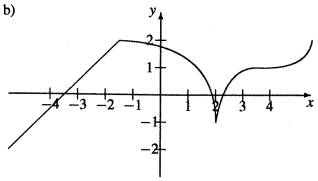


11.

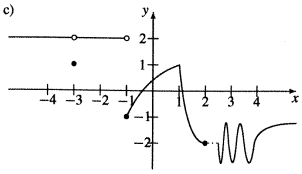




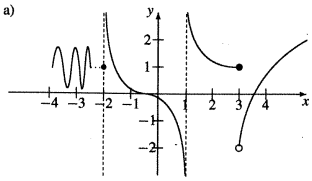
b)



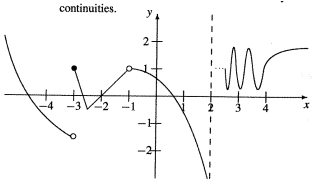
c)



a)



continuities.



4. Let

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$$

(a) $f(4) = 4$

(b) $\lim_{x \to 4^-} f(x) = 24$

(c) $\lim_{x \to 4^+} f(x) = 24$

(d) If f continuous?

(e) Can you change f slightly so that f is continuous?
(Can you redefine f at one point to do this?)

5. Let

$$f(x) = \frac{x^2 + 2x}{x + 2}$$

(a) What is the domain of f?

(b) $f(-2) = \text{DNE}$

(c) $\lim_{x \to -2^-} f(x) = -2$

(d) $\lim_{x \to -2^+} f(x) = -2$

(e) If f continuous?

(f) Can you change f slightly so that f is continuous?
(Can you redefine f at one point to do this?)
Solution: If you make $f(-2) = -2$ then f will be continous.

6. Let

$$f(x) = \begin{cases} 3x + 1 & \text{if } x < -1 \\ -2x - 4 & \text{if } x > -1 \end{cases}$$

(a) What is the domain of f?

(b) $f(-1) = \text{DNE}$

(c) $\lim_{x \to -1^-} f(x) = -2$

(d) $\lim_{x \to -1^+} f(x) = -2$

(e) If f continuous?

(f) Can you change f slightly so that f is continuous?
(Can you redefine f at one point to do this?)
Solution: If you make $f(-1) = -2$ then f will be continuous.

7. Let

$$f(x) = \begin{cases} 3x+1 & \text{if } x < -1 \\ -2x+4 & \text{if } x > -1 \end{cases}$$

(a) What is the domain of f?

(b) $f(-1) = \text{DNE}$

(c) $\lim_{x \to -1^-} f(x) = -2$

(d) $\lim_{x \to -1^+} f(x) = 6$

(e) If f continuous?

(f) Can you change f slightly so that f is continuous?
(Can you redefine f at one point to do this?)
Solution: No

8. Let

$$f(x) = \begin{cases} 3x+1 & \text{if } x \leq 1 \\ ax+1 & \text{if } x > 1 \end{cases}$$

(a) What is the domain of f?

(b) $f(1) = 4$

(c) $\lim_{x \to 1^-} f(x) = 4$

(d) $\lim_{x \to 1^+} f(x) = a+1$

(e) If f continuous?

(f) Can you find a so that f is continous?
Solution: Solve $4 = a+1$, which gives $a = 3$