Warm-up Problems

1. Compute
\[ \lim_{x \to -1} \frac{x^2 - 1}{|x| - 1} = 2 \]

2. Compute. Use a table, graph, etc.

   (a) \( \lim_{x \to 0^+} \frac{1}{x} = \infty \)
   (b) \( \lim_{x \to 0^-} \frac{1}{x} = -\infty \)
   (c) \( \lim_{x \to 0} \frac{1}{x} = \text{DNE} \)
   (d) \( \lim_{x \to 0^+} \frac{1}{x^2} = \infty \)
   (e) \( \lim_{x \to 0^-} \frac{1}{x^2} = \infty \)
   (f) \( \lim_{x \to 0} \frac{1}{x^2} = \infty \)

Lecture Problems

3. Suppose you know the following:
\[ \lim_{x \to 1} f(x) = 4 \]
\[ \lim_{x \to 1} g(x) = 7 \]
\[ \lim_{x \to 1} h(x) = 0 \]

Find the following

   (a) \( \lim_{x \to 1} f(x) \cdot g(x) = 28 \)
   (b) \( \lim_{x \to 1} f(x) \cdot h(x) = 0 \)
   (c) \( \lim_{x \to 1} f(x) + h(x) - g(x) = -3 \)
   (d) \( \lim_{x \to 1} f(x)/g(x) = 4/7 \)
   (e) \( \lim_{x \to 1} h(x)/g(x) = 0 \)
   (f) \( \lim_{x \to 1} f(x)/h(x) = \text{DNE} \)

4. Suppose you know that
\[ 2 - x^2 \leq g(x) \leq 2 \cos x \]
for all \( x \). Find \( \lim_{x \to 0} g(x) \).

Solution: Use the squeeze theorem to get 2
5. Find the limits

(a) \( \lim_{x \to 0} x^2 e^{\sin(1/x)} \)

**Solution:** Use squeeze theorem: 0

(b) \( \lim_{x \to 0} x^3 \sin(1/\sqrt[3]{x}) \)

**Solution:** Use squeeze theorem: 0

6. Play the *limit game* with a partner for the following limits.

(a) \( \lim_{x \to 5} 3x = 15 \)

(b) \( \lim_{x \to 3} -2x = -6 \)

(c) \( \lim_{x \to -1} 3x + 1 = -2 \)

(d) \( \lim_{x \to 1} 3x = 5 \)

(e) \( \lim_{x \to 13} -2x + 7 = 19 \)

(f) (More difficult) \( \lim_{x \to 1} x^2 = 1 \)

(g) (More difficult) \( \lim_{x \to 2} x^2 = 4 \)

(h) (More difficult) \( \lim_{x \to 2} x^2 - x + 1 = 3 \)