1. Use the steps discussed in class Monday to solve the differential equation:

\[ x^4 y' + 2x^3 y = 5 \quad y(1) = 4 \]

(1.) Write in standard form **Solution:** \( y' + \frac{2}{x} y = \frac{5}{x^4} \)

(2.) \( A(t) = \int \frac{2}{x} \, dx = 2 \ln x = \ln x^2 \)

\( I = e^A = e^{\ln x^2} = x^2 \)

(3.) Multiply the differential equation by the integrating factor \( I \)

**Solution:** \( x^2 y' + 2xy = \frac{5}{x^2} \quad \implies \quad (x^2 y)' = \frac{5}{x^2} \)

(4.) Integrate and solve for \( y \) (find a general solution):

**Solution:** \( x^2 y = \int \frac{5}{x^2} \, dx = -\frac{5}{x} + C \)

\( y = -\frac{5}{x^2} + \frac{C}{x^2} \)

(5.) If there is an initial condition use it and find a particular solution.

**Solution:** \( y = -\frac{5}{x^2} + \frac{9}{x^2} \)
2. Set up (and solve if you have time) the differential equations for the following problems.

(a) Let $y$ be the amount of money you have in the bank. You make no withdrawal or deposits, just let your money sit earning 5% annual interest compounded continuously. You start with $100 in the bank.

Set up the differential equation (with initial values).

Solution:

$$y' = 0.05y \quad y(0) = 100$$

(b) Let $y$ be the amount of money you have in the bank. You make continuous deposits of $500 per year. You earn 5% annual interest compounded continuously. You start with $0 in the bank.

Set up the differential equation (with initial values).

Solution:

$$y' = 0.05y + 500 \quad y(0) = 0$$

(c) Let $y$ be the amount of money you have in the bank. You make continuous withdrawals of $500 per year. You earn 5% annual interest compounded continuously. You start with $5,000 in the bank.

Set up the differential equation (with initial values).

Solution:

$$y' = 0.05y - 500 \quad y(0) = 5000$$

(d) Let $y$ be the amount of money you have in the bank. You make continuous withdrawals of $500 per year. You earn 5% annual interest compounded continuously. You start with $10,000 in the bank.

Set up the differential equation (with initial values).

Solution:

$$y' = 0.05y - 500 \quad y(0) = 10000$$
3. Set up (and solve if you have time) the differential equations for the following problems. Determine the steps to take to solve the problems.

(a) Let $y$ be the amount of money you have in the bank. You make no withdrawal or deposits, just let your money sit earning 5% annual interest compounded continuously. You start with $100 in the bank.

How long until you have $5000?

**Solution:** Solve the differential equation:

$$y' = 0.05y \quad y(0) = 100$$

To get $y = 100e^{0.05t}$.

To find when you will have $5000 you need so solve

$$5000 = y = 100e^{0.05t}$$

which gives you $t = 20 \ln 50 \approx 78.24$ (years).

(b) Let $y$ be the amount of money you have in the bank. You make continuous deposits of $500 per year. You earn 5% annual interest compounded continuously. You start with $0 in the bank.

How long until you have $5000?

**Solution:** Solve the differential equation:

$$y' = 0.05y + 500 \quad y(0) = 0$$

To get $y = 10000e^{0.05t} - 10000$.

To find when you will have $5000 you need so solve

$$5000 = y = 10000e^{0.05t} - 10000$$

which gives you $t = 20 \ln \left( \frac{3}{2} \right) \approx 8.11$ (years).
(c) Let $y$ be the amount of money you have in the bank. You make continuous withdrawals of $500 per year. You earn 5% annual interest compounded continuously. You start with $5,000 in the bank.

Will you run out of money? If so when?

**Solution:** Solve the differential equation:

$$y' = 0.05y - 500 \quad y(0) = 5000$$

To get $y = 10000 - 5000e^{0.05t}$.

To find when you will have $0 you need to solve

$$0 = 10000 - 5000e^{0.05t}$$

which gives you $t = 20 \ln 2 \approx 13.86$ (years)

(d) Let $y$ be the amount of money you have in the bank. You make continuous withdrawals of $500 per year. You earn 5% annual interest compounded continuously. You start with $10,000 in the bank.

Will you run out of money? If so when?

**Solution:** Solve the differential equation:

$$y' = 0.05y - 500 \quad y(0) = 10000$$

To get $y = 10000$.

Since this is a constant you will never change your money. Basically what is happening is your withdrawals are equal to the interest earned.