Warm-up Problems - January 22, 2006

Solutions

1. (a) Find all points of intersection for the lines

\[ y = x - 1, \quad y = -x + 2, \quad y = \frac{x}{2} - 1 \]

**Solution:** \((0, -1), \left(\frac{3}{2}, \frac{1}{2}\right), (2, 0)\).

(b) Find the area trapped between the lines

\[ y = x - 1, \quad y = -x + 2, \quad y = \frac{x}{2} - 1 \]

**Solution:**

\[
A = \int_0^{3/2} (x - 1) - \left(\frac{x}{2} - 1\right) \, dx + \int_{3/2}^2 (-x + 2) - \left(\frac{x}{2} - 1\right) \, dx
\]

\[= \frac{9}{16} + \frac{3}{16} = \frac{3}{4}\]

2. Let \(f(x) = x^2\).

(a) Find the average of \(f(1), f(2)\) and \(f(3)\).

**Solution:** Ave = \(\frac{14}{3}\)

(b) Find the average of \(f(.5), f(1), f(1.5), f(2), f(2.5), f(3)\).

**Solution:** Ave = \(\frac{91}{24} \approx 3.7917\)

(c) Find the average of \(f(.25), f(.5), f(.75), f(1), f(1.25), f(1.5), f(1.75), f(2), f(2.25), f(2.5), f(2.75), f(3)\).

**Solution:** Ave = \(\frac{325}{96} \approx 3.385417\)
**Lecture Problems**

3. A country’s population over the decade from 1990-2000 can be modelled by the function

\[ P = 3e^{0.02t} \]

where \( P \) is in millions and \( t \) is the number of years past 1990. Find the average population over the 1990’s decade.

\[
\text{Ave} = \frac{1}{10} \int_{0}^{10} 3e^{0.02t} \, dt = \frac{1}{10} \left[ \frac{3e^{0.02t}}{0.02} \right]_{0}^{10} \\
= \frac{1}{10} \left( \frac{3e^2}{0.02} - \frac{3}{0.02} \right) \approx 3.321 \text{ (Million)}
\]