Integration by parts:
\[ \int u \, dv = uv - \int v \, du \]

1. Each of the integrals below can be integrated using integration by parts. For each integral, determine the “correct” choice for \( u \) and \( dv \).

(a) \( \int xe^x \, dx \)
   \[ \text{Solution: } u = x, \, dv = e^x \, dx \]

(b) \( \int x \sin x \, dx \)
   \[ \text{Solution: } u = x, \, dv = \sin x \, dx \]

(c) \( \int x^2 \sin x \, dx \)
   \[ \text{Solution: } u = x^2, \, dv = \sin x \, dx. \text{ This will need to be integrated by parts twice.} \]

(d) \( \int x^2 e^x \, dx \)
   \[ \text{Solution: } u = x^2, \, dv = e^x \, dx. \text{ This will need to be integrated by parts twice.} \]

(e) \( \int \ln x \, dx \)
   \[ \text{Solution: } u = \ln x, \, dv = dx \]

(f) \( \int x^2 \ln x \, dx \)
   \[ \text{Solution: } u = \ln x, \, dv = x^2 \, dx \]

(g) \( \int x \cos x \, dx \)
   \[ \text{Solution: } u = x, \, dv = \cos x \, dx \]
2. Compute the integrals in Problem 1.

Solution:

(a) \[ \int xe^x \, dx = (x - 1)e^x + C \]

(b) \[ \int x \sin x \, dx = \sin x - x \cos x + C \]

(c) \[ \int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x + C \]

(d) \[ \int x^2 e^x \, dx = (x^2 - 2x + 2)e^x + C \]

(e) \[ \int \ln x \, dx = x \ln x - x + C \]

(f) \[ \int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \frac{x^3}{9} + C \]

(g) \[ \int x \cos x \, dx = x \sin x + \cos x \]
Lecture Problems

3. Approximate the integrals using a left hand sum, right hand sum, midpoint rule, trapezoid rule and Simpson’s rule. Use $\Delta x = \frac{1}{2}$.

(a)

$$\int_{0}^{1} e^{-x} \, dx = 1 - e^{-1} \approx 0.6321205588$$

L = 0.8032653298
R = 0.4872050504
M = 0.6255836680
T = 0.6452351901
S = 0.6321341753

(b)

$$\int_{0}^{2} \sqrt{1 + x^3} \, dx \approx 3.241309265$$

L = 2.783261900
R = 3.783261900
M = 3.220226115
T = 3.283261900
S = 3.241238043