Warm-up Problems - February 14, 2007
Solutions

1. Find the limits for given region for the double integral $\int \int_{R} f(x, y) \, dx \, dy$

(a) Let $R$ be the region in the first quadrant bounded by the $x$-axis, $x = 4$ and $y = x$.

$$\int \int_{R} f(x, y) \, dy \, dx = \int_{0}^{4} \int_{0}^{x} f(x, y) \, dy \, dx$$

(b) Let $R$ be the region in the first quadrant bounded by the $y = x^2$ and $y = x$.

$$\int \int_{R} f(x, y) \, dy \, dx = \int_{0}^{1} \int_{x^2}^{x} f(x, y) \, dy \, dx = \int_{0}^{1} \int_{\sqrt{y}}^{y} f(x, y) \, dx \, dy$$

(c) Let $R$ be the region bounded by $y = x^2$ and $y = 4$

$$\int \int_{R} f(x, y) \, dy \, dx = \int_{-2}^{2} \int_{x^2}^{4} f(x, y) \, dy \, dx$$
(d) Let \( R \) be the region in the first quadrant bounded by the \( y \)-axis, \( y = x \) and \( y = 2 - x \)

\[
\int\int_{R} f(x, y) \, dy \, dx = \int_{0}^{1} \int_{x}^{2-x} f(x, y) \, dy \, dx \\
= \int_{0}^{1} \int_{y}^{2-y} f(x, y) \, dx \, dy + \int_{1}^{2} \int_{0}^{2-y} f(x, y) \, dx \, dy
\]
Lecture Problems

2. Fill out the chart below

<table>
<thead>
<tr>
<th>Radians</th>
<th>$\frac{-\pi}{6}$</th>
<th>$\frac{5\pi}{12}$</th>
<th>$\frac{13\pi}{60}$</th>
<th>$\frac{23\pi}{180}$</th>
<th>$\frac{\pi}{4}$</th>
<th>$\frac{\pi}{6}$</th>
<th>$\frac{\pi}{3}$</th>
<th>0</th>
<th>$\pi$</th>
<th>$2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees</td>
<td>$-30^\circ$</td>
<td>$75^\circ$</td>
<td>$411^\circ$</td>
<td>$23^\circ$</td>
<td>$45^\circ$</td>
<td>$30^\circ$</td>
<td>$60^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. For each of the angles below:

(i) Find the radian measure and degree measure of the angle.

(ii) Draw the angle on the unit circle.

(iii) Determine if $\sin t$ and $\cos t$ and positive or negative.

(a) $t = \frac{\pi}{3}$

$60^\circ$, $\sin t > 0$, $\cos t > 0$

(b) $t = \frac{-\pi}{3}$

$-60^\circ$, $\sin t < 0$, $\cos t > 0$
(c) $t = -165^\circ$

\[-\frac{11\pi}{12}, \quad \sin t < 0, \quad \cos t < 0\]

(d) $t = 2$

\[\left(\frac{360}{\pi}\right)^\circ, \quad \sin t > 0, \quad \cos t > 0\]