Warm-up Problems - April 25, 2007

Topics for Final Exam:

I. Integration: definite integrals, indefinite integrals, area under curves, integration by substitution, integration by parts, improper integrals

II. Approximate integration: Simpson’s rule (nothing on errors)

III. Applications of integration: average value, present and future value of a continuous income stream. (NOT: consumer’s and producers surplus).

IV. Multivariable calculus: partial derivatives, graphing, level curves, maxima and minima, lagrange multipliers, least squares, double integrals over rectangles

V. Trigonometry: degrees and radians, computing trigonometric functions, derivatives and integrals.

VI. Differential equations: separation of variables, slope fields, first order linear, setting up equations, graphing autonomous equations.

VII. Taylor series: finding Taylor polynomials, finding Taylor series (NOT: Taylor reminder theorem).

VIII. Series: integral test, geometric series

IX. Newton method for finding roots.

X. Probability and statistics: using a pdf, finding a cdf, mean, median, variance, standard deviation, using exponential random variables, using normal random variables.

1. Compute the integrals
   \[ \int x^3 \ln x \, dx = \]
   \[ \int \sin(\cos x) \sin x \, dx = \]
   \[ \int \frac{3x^2}{15x^3 + 1} \, dx = \]

2. Approximate the following integrals using Simpson’s rule and \( n = 4 \).
   \[ \int_0^2 e^{x^2} \, dx \approx \]
   \[ \int_1^3 \sin(x^2) \, dx \approx \]

3. Solve the initial value problems
   \[ y' = xy^2, y(0) = 2. \]
   \[ y' = \frac{y}{x} + 1, y(1) = 2. \]
4. Find the least squares line for the data

(a)

\[
\begin{array}{cccc}
 x & 1 & 2 & 4 & 8 \\
 y & 1 & 5 & 6 & 6 \\
\end{array}
\]

(b)

\[
\begin{array}{cccc}
 x & 1 & 2 & 4 & 8 \\
 y & -1 & 5 & 23 & -16 \\
\end{array}
\]

5. A continuous income stream produces income for 10 years at a rate of \( f(t) = 100 + t \) dollars per year. Interest is 10% per year.

(a) Find the income that is produced by this income stream (total income).

(b) Find the future value of this income stream

(c) Find the present value of this income stream

6. Find the integrals

(a) \( \int_0^\infty e^{-2x} \, dx = \)

(b) \( \int_1^\infty \frac{\ln x}{x} \, dx = \)

(c) \( \int_1^\infty \frac{\ln x}{x^2} \, dx = \)

7. Find the Taylor series

(a) \( f(x) = \frac{1}{(1 - x)^3} \) at \( x = 0 \)

(b) \( f(x) = \frac{1}{1 - x^3} \) at \( x = 0 \)

8. Find the average value of the functions

(a) Average of \( f(x) = \sin x \) on \([0, \pi]\)

(b) Average of \( f(x) = \cos x \) on \([0, \pi]\)

9. Find the sum of the series

(a) \( 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots = \)

(b) \( 1 + 2 + 2^2 \frac{2^2}{2!} + 2^3 \frac{2^3}{3!} + 2^4 \frac{2^4}{4!} + 2^5 \frac{2^5}{5!} + \cdots = \)