Warm-up Problems - April 25, 2007
Solutions

1. Compute the integrals
   (a) \( \int x^3 \ln x \, dx = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C \)
   (b) \( \int \sin(\cos x) \sin x \, dx = \cos(\cos(x)) + C \)
   (c) \( \int \frac{3x^2}{15x^3 + 1} \, dx = \frac{1}{15} \ln(15x^1 + 1) + C \)

2. Approximate the following integrals using Simpson’s rule and \( n = 4 \).
   (a) \( \int_0^2 e^{x^2} \, dx \approx 16.53859470 \)
   (b) \( \int_1^3 \sin(x^2) \, dx \approx 0.4683011405 \)

3. Solve the initial value problems
   (a) \( y' = xy^2, y(0) = 2. \)
      Solution: \( y = -\frac{2}{x^2 - 1} \)
   (b) \( y' = \frac{y}{x} + 1, y(1) = 2. \)
      Solution: \( y = x(\ln x + 2) \)

4. Find the least squares line for the data
   (a)
   \[
   \begin{array}{cccc}
   x & 1 & 2 & 4 & 8 \\
   y & 1 & 5 & 6 & 6 \\
   \end{array}
   \]
   Solution: \( y = \frac{62}{115} x + \frac{57}{23} \)
   (b)
   \[
   \begin{array}{cccc}
   x & 1 & 2 & 4 & 8 \\
   y & -1 & 5 & 23 & -16 \\
   \end{array}
   \]
   Solution: \( y = -\frac{273}{115} x + \frac{268}{23} \)
5. A continuous income stream produces income for 10 years at a rate of \( f(t) = 100 + t \) dollars per year. Interest is 10% per year.

(a) Find the income that is produced by this income stream (total income).

**Solution:** \( TI = 1050 \)

(b) Find the future value of this income stream

**Solution:** \( FV = 1790.11 \)

(c) Find the present value of this income stream

**Solution:** \( PV = 658.54 \)

6. Find the integrals

(a) \[ \int_{0}^{\infty} e^{-2x} \, dx = \frac{1}{2} \]

(b) \[ \int_{1}^{\infty} \frac{\ln x}{x} \, dx = \text{diverges} \]

(c) \[ \int_{1}^{\infty} \frac{\ln x}{x^2} \, dx = 1 \]

7. Find the Taylor series

(a) \( f(x) = \frac{1}{(1-x)^3} \) at \( x = 0 \)

**Solution:** \[ \sum_{n=0}^{\infty} \frac{(n+2)(n+1)}{2} x^n \]

(b) \( f(x) = \frac{1}{1-x^3} \) at \( x = 0 \)

**Solution:** \[ \sum_{n=0}^{\infty} x^{3n} \]

8. Find the average value of the functions

(a) Average of \( f(x) = \sin x \) on \([0, \pi]\)

**Solution:** \( \text{ave} = \frac{2}{\pi} \)

(b) Average of \( f(x) = \cos x \) on \([0, \pi]\)

**Solution:** \( \text{ave} = 0 \)

9. Find the sum of the series

(a) \[ 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots = e \]

(b) \[ 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \cdots = e^2 \]