Warm-up Problems - April 22, 2007

Solutions

1. Let $Z$ be the standard normal random variable. Use the chart on the back to compute the probabilities. 
   (Hint: Make sure you draw the area that the question is asking for!)
   
   (a) $P(-1.21 \leq Z \leq 0.54) = 0.3869 + 0.2054 = 0.5923$
   (b) $P(-1.21 \leq Z \leq 1.21) = 0.3869 + 0.3869 = 0.7738$
   (c) $P(0.54 \leq Z \leq 1.21) = 0.3869 - 0.2054 = 0.1815$
   (d) $P(Z \geq 3) =$

Lecture Problems

2. The lengths of trout in a certain lake are normally distributed with a mean of 7 inches and standard deviation of 2 inches.

   (a) What proportion of trout are longer than 8 inches?
   **Solution:** Convert to $z$. $X = 8 \implies Z = (8 - 7)/2 = 1/2$.
   
   $P(X > 8) = P(Z > 0.5) = 0.5 - 0.1915 = 0.3085$
   
   (b) What proportion of trout are shorter than 6 inches?
   **Solution:** Convert to $z$. $X = 6 \implies Z = (6 - 7)/2 = -1/2$.
   
   $P(X < 6) = P(Z < -0.5) = 0.5 - 0.1915 = 0.3085$
   
   (c) What proportion of trout are between 6 and 8 inches?
   **Solution:** Convert to $z$ (done above).
   
   $P(6 \leq X \leq 8) = P(-0.5 \leq Z \leq 0.5) = 2(0.1915) = 0.3830$
   
   (d) Suppose the fish and game department would like anglers to only keep the largest 20% of the trout. What should the minimum size for “keepers” be?
   **Solution:**
   
   i. Find the $z$ score that corresponds to 0.3. This is the $z$ so that
   $P(Z \geq z) = 0.2 = 0.5 - P(0 \leq Z \leq z)$
   We look on the chart and find $z = 0.84$.
   
   ii. Convert this $z$ into an $x = z\sigma + \mu = (0.84)(2) + 7 = 8.68$ (inches)
   
   (e) Suppose they change thier minds and the fish and game department would like anglers to only keep the largest 8% of the trout. What should the minimum size for “keepers” be?
   **Solution:**
   
   i. Find the $z$ score that corresponds to 0.42. This is the $z$ so that
   $P(Z \geq z) = 0.08 = 0.5 - P(0 \leq Z \leq z)$
   We look on the chart and find $z = 1.405$.
   
   ii. Convert this $z$ into an $x = z\sigma + \mu = (1.405)(2) + 7 = 9.81$ (inches)
3. The amount of garbage a household in the U.S. produce is a normal distribution with mean 225 pounds and standard deviation of 19 pounds.

(a) What percentage of households discard more than 250 pounds of garbage?

\textbf{Solution:} \\
\[ P(X > 250) = P(Z > 1.32) = 0.5 - 0.4066 = 0.0934 \]

(b) What percentage of households discard less than 100 pounds of garbage?

\textbf{Solution:} \\
\[ P(X < 100) = P(Z < -6.58) = 0 \]

(c) What percentage of households discard between 250 and 300 pounds of garbage?

\textbf{Solution:} \\
\[ P(250 < X < 300) = P(1.32 < Z < 3.95) = 0.5 - 0.4066 = 0.0934 \]

(d) Find the weight of garbage discarded by the households that produce the highest 5\% of garbage.

\textbf{Solution:} Find the \( z \) value that corresponds to 0.45, \( z = 1.645 \). The corresponding \( x \) value is \( x = z\sigma + \mu = 256.3 \) pounds.

(e) Find the weight of garbage discarded by the households that produce the lowest 10\% of garbage.

\textbf{Solution:} Find the \( z \) value that corresponds to 0.40, \( z = 1.28 \). We actually need to use \( z = -1.28 \) (draw the graph and see what areas are in question). The corresponding \( x \) value is \( x = z\sigma + \mu = 200.7 \) pounds.