Warm-up Problems - April 4, 2007
Solutions

1. For the following series, find the sum if they converge (they are all geometric)

(a) \[ \sum_{n=0}^{\infty} \frac{5(2^n)}{3^n} = \frac{5}{1 - 2/3} = 15 \quad (a = 5, r = \frac{2}{3}) \]

(b) \[ \sum_{n=2}^{\infty} \frac{5(2^n)}{3^n} = \frac{20/9}{1 - 2/3} = \frac{20}{3} \quad (a = \frac{20}{9}, r = \frac{2}{3}) \]

(c) \[ \sum_{n=0}^{\infty} \frac{5(3^n)}{2^n} = \text{diverges since } r > 1 \quad (a = 5, r = \frac{3}{2}) \]

(d) \[ \sum_{n=0}^{\infty} \frac{2^n + 3^n}{7^n} = \sum_{n=0}^{\infty} \frac{2^n}{7^n} + \sum_{n=0}^{\infty} \frac{3^n}{7^n} = \frac{1}{1 - 2/7} + \frac{1}{2 - 3/7} = \frac{7}{5} - \frac{7}{4} \]
Lecture Problems

2. Determine if the following series converge or diverge

(a)
\[ \sum_{n=1}^{\infty} \frac{1}{n^7} \]
\[ \int_{1}^{\infty} \frac{1}{x^7} \, dx = \frac{1}{6} \text{ converges by integral test} \]

(b)
\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} \]
\[ \int_{1}^{\infty} \frac{1}{x^{1/3}} \, dx = \infty \text{ diverges by integral test} \]

(c)
\[ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \]
\[ \int_{1}^{\infty} \frac{1}{x \ln x} \, dx = \infty \text{ diverges by integral test} \]

(d)
\[ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \]
\[ \int_{2}^{\infty} \frac{1}{x(\ln x)^2} \, dx = \frac{1}{\ln 2} \text{ diverges by integral test} \]
Topics for Exam 3:

1. First order linear differential equations.

2. Applications of differential equations
   (a) Financial models (rate in minus rate out)
   (b) Logistic population model (need to understand graph but do not need to solve explicitly)
   (c) Mixing problems (setup and need to understand graph but do not need to solve explicitly)

3. Graphing solutions to autonomous differential equations (equations without the $t$)

4. Taylor polynomials (find any degree Taylor polynomial for any function centered at any point)
   (a) Making a “derivative” chart for the function and using the chart to find the Taylor polynomials
   (b) Using the Taylor series for known functions and plugging in (or other tricks we’ve discussed).
      The polynomials you should “know” are $e^x$, $\frac{1}{1-x}$, $\sin x$, $\cos x$.

5. Remainder theorem for Taylor Polynomials—find a good estimate for the error of your polynomial.

6. Taylor series—find the general term (mostly this is just like the material on Taylor polynomials)

7. Geometric Series (determining if series converges or diverges, finding the sum)

8. Integral test for convergence of series with positive terms. (No alternating series, no comparison test.)