Warm-up Problems - March 8, 2006
Solutions

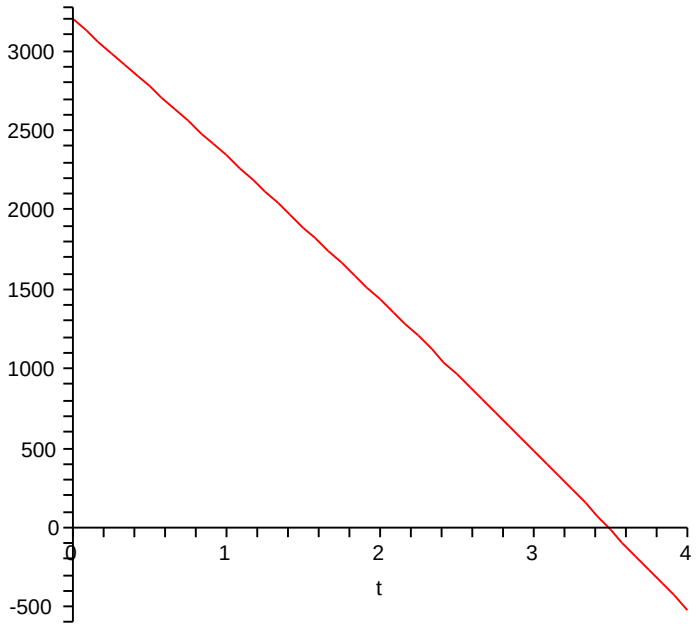
1. Solve the first order linear differential equation.

   1) Write the DE as $y' + f(x)y = g(x)$.
   2) Find the integrating factor, $I = e^{\int f(x)\,dx}$
   3) Multiply the differential equation by I.
   4) Recognize the product rule in your differential equation.
   5) Integrate your DE, solve.
   6) Use initial condition.

   (a) $P' = 0.05P - 1000$, $P(0) = 3200$

   Solution:

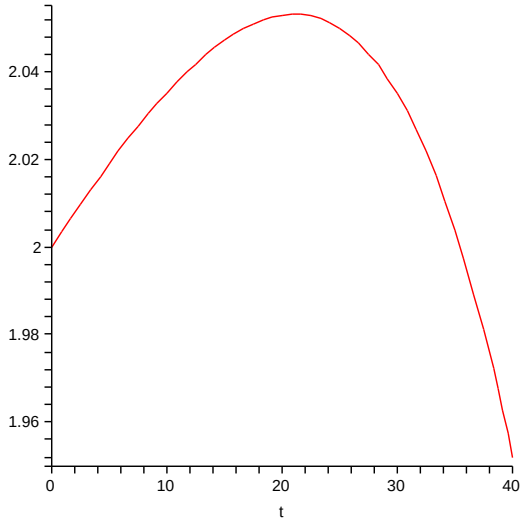
$$\begin{aligned} I =& e^{-0.05t} \\ P =& 20000 + Ce^{0.05t} \\ P =& 20000 - 16800e^{0.05t} \end{aligned}$$

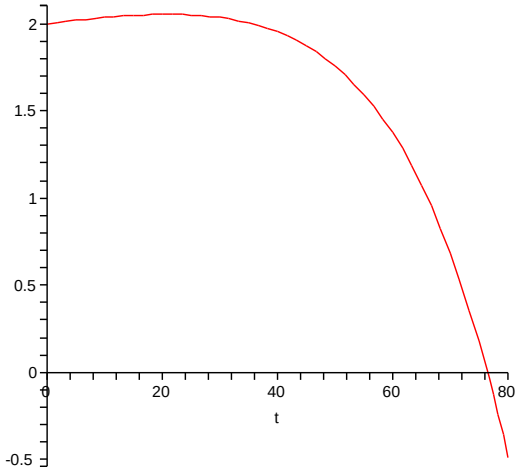


(b) $y' = \frac{3}{125}y - 0.004e^{0.04t} - 0.04,\ y(0) = 2$

Solution:

$$
\begin{aligned}
I =& e^{-3t/125} \\
e^{-3t/125}y' - \frac{3}{125}e^{-3t/125}y =& e^{-3t/125}(-0.004e^{0.04t} - 0.04) \\
\left(e^{-3t/125}y\right)' =& -\frac{1}{250}e^{2t/125} - \frac{1}{25}e^{-3t/125} \\
e^{-3t/125}y =& -\frac{1}{4}e^{2t/125} + \frac{5}{3}e^{-3t/125} + C \\
y =& -\frac{1}{4}e^{t/25} + \frac{5}{3} + Ce^{3t/125} \\
y =& -\frac{1}{4}e^{t/25} + \frac{5}{3} + \frac{7}{12}e^{3t/125}
\end{aligned}
$$





(c) $y' = 6 - \frac{2y}{50+t}$, $y(0) = 20$.

Solution:

$$\begin{aligned} y' + \frac{2y}{50+t} &= 6 \\ I &= (50+t)^2 \\ (50+t)^2 y' + 2(50+t)y &= 6(50+t)^2 \\ \left[(50+t)^2 y\right]' &= 6(50+t)^2 \\ (50+t)^2 y &= 2(50+t)^3 + C \\ y &= 2(50+t) + \frac{C}{(50+t)^2} \\ y &= 2(50+t) - \frac{200,000}{(50+t)^2} \end{aligned}$$