Warm-up Problems - March 6, 2006
Solutions

1. A consulting firm arrived at the following Cobb-Douglass function for a particular product:

   $$N(x, y) = 50x^{0.7}y^{0.3}$$

   where x is the number of units of capital and y is the number of units of labor.

   Each unit of capital costs \$40 and each unit of capital costs \$80. The company has \$400,000 to invest. How should the money be allocated to maximize production?

   (Use Lagrange multipliers)

   Solution: Make the function

   $$F(x, y, \lambda) = 50x^{0.7}y^{0.3} + \lambda(40x + 80y - 400,000)$$

   Now solve $F_x = 0$, $F_y = 0$, $F_\lambda = 0$. Solving gives

   $$\begin{aligned} x =& 7000 \\ y =& 1500 \end{aligned}$$

2. If instead, the price of capital goes up to \$60 a unit, how should the investment money be allocated.

   (Use Lagrange multipliers)

   Solution: In this case, make the function

   $$F(x, y, \lambda) = 50x^{0.7}y^{0.3} + \lambda(60x + 80y - 400,000)$$

   Now solve $F_x = 0$, $F_y = 0$, $F_\lambda = 0$. Solving gives

   $$\begin{aligned} x =& \frac{14000}{3} \approx 4666.67 \\ y =& 1500 \end{aligned}$$

Lecture Problems

3. For each of the following first order linear differential equations:

   1) Write the DE as $y' + f(x)y = g(x)$.
   2) Find the integrating factor, $I = e^{\int f(x)\,dx}$
   3) Multiply the differential equation by I.
   4) Recognize the product rule in your differential equation.
   5) Integrate your DE, solve.
   6) Use initial condition (if any).

   (a) $2y' + 4y = 1$

   $$\begin{aligned} I =& e^{2x} \\ y =& \frac{1}{4} + Ce^{-2x} \end{aligned}$$

   (b) $y' = y - x$

   $$\begin{aligned} I =& e^{-x} \\ y =& 1 + x + ce^{x} \end{aligned}$$

   (c) $y' - \frac{y}{x} = 2$

   $$\begin{aligned} I =& \frac{1}{x} \\ y =& 2x \ln x + Cx \end{aligned}$$

   (d) $y' + y = x - 1$

   $$\begin{aligned} I =& e^{x} \\ y =& -2 + x + Ce^{-x} \end{aligned}$$

   (e) $y' + x^2 y = x^2$
   Solution:

   $$\begin{aligned} I =& e^{x^3/3} \\ y =& 1 + Ce^{-x^3/3} \end{aligned}$$