Warm-up Problems - March 31, 2006

1. Concept Review:
   (a) First order linear differential equations
      i. "Standard form"
      ii. Initial condition
      iii. Integrating factor
   (b) Modeling with differential equations
      i. Money, interest
      ii. Mixture
   (c) Taylor polynomials and series
      i. Finding Taylor polynomials (at $x = 0$ and at other points)
      ii. Using Taylor polynomials to approximate functions
      iii. Formula for the Taylor series (what is the general formula for a_n?)
      iv. Finding Taylor series using series you know (which series should you know?)
      v. Taylor's formula for the remainder.
      vi. Alternating series formula for remainder.

2. Solve the initial value problem

   $$t^2y' + ty = 2, \qquad y(1) = 1$$

3. You take out a car loan for \$25,000 with interest rate 5% per year. You make payments at a rate of \$4800 per year.

   Set up a differential equation modeling this situation. How long does it take to pay off the loan?

4. We start with a tank containing 50 gallons of salt water with the salt concentration being 2 pounds per gallon. Salt water with a salt concentration of 3 pounds per gallon is then poured into the top of the tank at the rate of 5 gal/min. Salt water is at the same time drained from the bottom of the tank at the rate of 4 gal/min.

   How much salt will be in the tank after an hour?

   Let y be the amount of salt in the tank. Set up the differential equation, solve it, etc.

5. Let $f(x) = e^x$. Using the 5th degree Taylor polynomial make the approximations. Estimate the error in each case.

   (a) e^{-1}

   (b) $e^{-1/2}$

   (c) $e^{1/2}$ (find something useful for M, $e^{1/2}$ isn't good)

6. Find a Taylor series for $\dfrac{1}{(1-x)^2}$.

7. Find a Taylor series for $\dfrac{1}{(1-x)^3}$.

8. Using the second degree taylor polynomials in the previous problems, find an approximation for the integrals:

   (a)

   $$\int_0^{1/2} \frac{1}{(1-x)^2}\,dx \approx$$

   (b)

   $$\int_0^{1/2} \frac{1}{(1-x)^3}\,dx \approx$$

9. Find a Taylor series for $\dfrac{1}{(1+x)^3}$.

10. Use a Taylor series to approximate the integral below to within 0.001.

    What is the degree of the Taylor polynomial needed to make this

    $$\int_0^{1/4} \frac{1}{(1+x)^3}\,dx$$