Warm-up Problems - March 3, 2006
Solutions

1. Identify each of the following

    1) Partial derivatives
    2) Second derivative test
    3) Lagrange multipliers
    4) Constraint, multiple constraints
    5) Method of least squares (for any function, not just a line)
    6) Residuals
    7) Formula for least square line
    8) Double integrals
    9) Average value of a function
    10) Solutions to differential equations
    11) Slope fields
    12) General solution, particular solution
    13) Initial value problem, initial condition
    14) Separation of variables

Lecture Problems

2. Let

$$f(x, y) = \frac{x - 2y}{x^2 + 2y}$$

Find f_y at the point $(1, 2)$.

Solution: $\dfrac{-4}{25}$

3. Find the minimum of the function $f(x, y) = 2x^2 - 2xy + 3y^2 - 4x - 8y + 20$

Solution: Only critical point is at $(2, 2)$, which by second derivative test is a minimum of 8.

4. Find and classify the critical points of the function $f(x, y) = x^2 + y^2 - 3xy$

Solution: Saddle at $(0, 0)$, local minimum at $(1, 1)$.

5. For the data below, find the least squares regression line and predict a value for $y(10)$.

x	2	4	7	9
y	0	1	5	8

Solution: $y = 1.172x - 2.948$, $y(10) = 8.77$.

6. Find the average value of the function $f(x, y) = x^2y^3$ on the rectangle $0 \le x \le 2$, $-1 \le y \le 1$.

Solution: Average value is 0.

7. Find the minimum of $f(x, y) = x^2 + 2y^2$ subject to the constraint $x - 2y = 3$.

Do this 2 ways. Using Lagrange multipliers and without.

Solution: Min: 3 at $(1, -1)$.

Using Lagrange multipliers, $\lambda = -2$

8. It is projected that $N(t)$, the total number of a new product sold, will grow in a way described by the equation

$$\frac{dN}{dt} = 0.1(M - N)$$

where M is some unspecified constant. After 10 months, it is found that $N(10) = 600$. What is the projected value for $N(20)$?

Solution: 821