Warm-up Problems - March 23, 2006
Solutions

1. For each of the functions below, find the Taylor polynomials at $x = 0$ p_0, p_1, p_2 and p_3

   (a) $f(x) = \frac{100}{x+10} + 10$

   Solution:

n	$f^{(n)}(x)$	$f^{(n)}(0)$	a_n
0	$\dfrac{100}{x+10} + 10$	20	20
1	$-\dfrac{100}{(x+10)^2}$	-1	-1
2	$\dfrac{200}{(x+10)^3}$	$\frac{1}{5}$	$\frac{1}{10}$
3	$-\dfrac{600}{(x+10)^4}$	$-\frac{3}{50}$	$-\frac{1}{100}$

   $$p_3(x) = 20 - x + \frac{1}{10}x^2 - \frac{1}{100}x^3$$

   (b) $f(x) = \frac{100}{x^2+10} + 10$ (Challenge: find p_4 and p_6)

   Solution:

n	$f^{(n)}(x)$	$f^{(n)}(0)$	a_n
0	$\dfrac{100}{x^2+10} + 10$	20	20
1	$-\dfrac{200x}{(x^2+10)^2}$	0	0
2	$\dfrac{200(3x^2-10)}{(x^2+10)^3}$	-2	-1
3	$-\dfrac{2400x(x^2-10)}{(x^2+10)^4}$	0	0
4	$\dfrac{12000(x^4-20x^2+20)}{(x^2+10)^5}$	$\frac{12}{5}$	$\frac{1}{10}$
5	$\dfrac{24000x(3x^4-100x^2+300)}{(x^2+10)^6}$	0	0
6	$\dfrac{72000(7x^6-350x^4+2100x^2-1000}{(x^2+10)^7}$	$-\frac{36}{5}$	$-\frac{1}{100}$

   $$p_6(x) = 20 - x^2 + \frac{1}{10}x^4 - \frac{1}{100}x^6$$

Lecture Problems

2. Find the Taylor polynomials p_0, p_1, p_2, p_3 for $f(x) = e^x$ at $x = 2$.

   Solution:

   $$p_3 = e^2 + e^2(x-2) + \frac{e^2}{2}(x-2)^2 + \frac{e^2}{6}(x-2)^3$$

3. Find the Taylor polynomials p_0, p_1, p_2, p_3 for $f(x) = e^{2x}$ at $x = 1$.

   Solution:

   $$p_3 = e^2 + 2e^2(x-1) + 2e^2(x-2)^2 + \frac{4e^2}{3}(x-2)^3$$

4. Find the Taylor polynomials p_0, p_1, p_2, p_3 for $f(x) = \sqrt{x}$ at $x = 1$.

   Solution:

   $$p_3 = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-2)^2 + \frac{1}{16}(x-2)^3$$

5. Find the Taylor polynomials p_0, p_1, p_2, p_3 for $f(x) = \sqrt{1+x}$ at $x = 8$.

   Solution:

   $$p_3 = 3 + \frac{1}{6}(x-8) - \frac{1}{216}(x-8)^2 + \frac{1}{3888}(x-8)^3$$