

Warm-up Problems - March 1, 2006

Solutions

1. Show that the general solution given is really a solution to the differential equation. Then, solve the initial value problem.

(a) $y' = 5y + 1, y(0) = 1$
 $y = -\frac{1}{5} + Ce^{5x}$

Solution:

$$y = -\frac{1}{5} + \frac{6}{5}e^{5x}$$

(b) $y' = y - x, y(0) = 1$
 $y = 1 + x + Ce^x$

Solution:

$$y = 1 + x$$

(c) $y' = y - x, y(0) = 2$
 $y = 1 + x + Ce^x$

Solution:

$$y = 1 + x + e^x$$

(d) $y' = y - x, y(1) = 4$
 $y = 1 + x + Ce^x$

Solution:

$$y = 1 + x + \frac{2}{e}e^x$$

Lecture Problems

2. Solve the initial values problems

(a) $\frac{dy}{dx} = x^2 + 1, y(0) = -4$

Solution:

General solution: $y = \frac{x^3}{3} + x + C$

Particular solutions: $y = \frac{x^3}{3} + x - 4$

(b) $\frac{dy}{dx} = y^2 x^2 + y^2, y(0) = -4$

Solution:

General solution: $-\frac{1}{y} = \frac{x^3}{3} + x + C$ or $y = -\frac{1}{x^3/3 + x + C}$

Particular solutions: $y = -\frac{1}{x^3/3 + x + 1/4}$

(c) $\frac{dy}{dx} = x^2 y + y, y(0) = -4$

Solution:

General solution: $\ln |y| = \frac{x^3}{3} + x + C$ or $y = Ae^{\frac{x^3}{3} + x}$

Particular solutions: $y = -4e^{\frac{x^3}{3} + x}$

3. Write down a differential equation representing the situation described below. Make sure you say what all your variables are.

(a) Sales of a certain product are declining at a rate proportional to the amount of sales.

Solution:

$$\frac{dS}{dt} = kS$$

(b) A certain fish population is limited to 5000 by the amount of food available. The population of the fish is growing at a rate proportional the difference between this maximum population and the actual population.

Solution:

$$\frac{dP}{dt} = k(5000 - P)$$

(c) Your grandma just baked a pie and set it out to cool. The rate of change of the temperature of the pie is proportional to the difference between the temperature of the pie and the temperature of the room that pie is in.

Solution:

$$\frac{dT}{dt} = k(T - R)$$