1. Compute the following integrals

   (a) $\int \frac{\ln x}{x} \, dx$

   **Solution:** This is an integration by substitution with $u = \ln x$:

   $$\int \frac{\ln x}{x} \, dx = \int u \, du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$$

   (b) $\int \frac{\ln x}{x^2} \, dx$

   **Solution:** This is an integration by parts. I choose $u = \ln x$ and $dv = x^{-2} \, dx$:

   $$\int \frac{\ln x}{x^2} \, dx = -\frac{\ln x}{x} - \int -1 \cdot \frac{1}{x} \, dx = -\frac{\ln x}{x} + \int \frac{1}{x^2} \, dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

2. Find the number $a > 0$ so that $f(x)$ below is a probability density function:

   $$f(x) = \begin{cases} 
   x^2 & \text{if } 0 \leq x \leq a \\
   0 & \text{otherwise}
   \end{cases}$$

   **Solution:** Remember that a pdf must satisfy $f(x) \geq 0$ (this is not an issue here) and $\int_{-\infty}^{\infty} f(x) \, dx = 1$

   So, we just have to find $a$ so that $\int_{0}^{a} x^2 \, dx = 1$.

   This gives the equation $\frac{a^3}{3} = 1$ or $a = 3^{1/3}$. 
Lecture Problems

3. Graph the following in $\mathbb{R}^3$:

(a) $x = 1$

(b) $x^2 + y^2 = 1$

(c) $y = e^x$

4. Let $z = f(x, y) = x + y^2$. Describe the cross sections and graph.

(a) $x = 0$
   
   **Solution:** This is a parabola $z = y^2$

(b) $x = 1$
   
   **Solution:** This is a parabola $z = y^2 + 1$

(c) $x = -1$
   
   **Solution:** This is a parabola $z = y^2 - 1$

(d) $y = 0$
   
   **Solution:** This is a plane $z = x$

(e) $y = 1$
   
   **Solution:** This is a plane $z = x + 1$