1. The formula for future values of a continuous income stream is:

\[ FV = e^{rt} \int_{0}^{T} f(t)e^{-rt} \, dt \]

Explain what each term stands for:

(a) \( f(t) \) 
Solution: This is the flow rate—how much money is the income stream producing?

(b) \( r \) 
Solution: This is the interest rate that the investment is earning. Remember after earning the money from the income stream, the money is invested with continuous interest.

(c) \( T \) 
Solution: This is the amount of time that you wait—how long does the income stream produce money and how long does the invested money sit there?

(d) \( FV \) (what does future value mean?) 
Solution: Future value is how much

2. What is the equation \( FV = Pe^{rt} \)? Why isn’t this used for an income stream? (In what situations is this equation used?)

Solution: This equation is when a single lump sum of money is invested with an interest rate of \( r \), compounded continuously. Once the money is invested, nothing is done with it for \( t \) years. This is different from an income stream since in an income stream we keep investing what we earn, as we earn it.

3. Find the following integrals:

(a) \( \int \frac{x^2}{x^3+1} \, dx \)
Solution: \( \frac{1}{3} \ln(x^3 + 1) \)

(b) \( \int xe^{7x^2+5} \, dx \)
Solution: \( \frac{1}{14} e^{7x^2+5} \)
Lecture Problems

4. Find the Gini index for the Lorenz curve \( f(x) = xe^{x-1} \).

Solution:

\[
\text{Ind} = 2 \int_0^1 x - xe^{x-1} \, dx = 2 \left[ \frac{x^2}{2} - (x - 1)e^{x-1} \right]_0^1
\]
\[
= 2 \left[ \left( \frac{1}{2} - 0 \right) - (0 + e^{-1}) \right] = 1 - 2e^{-1} \approx 0.264
\]

5. For each of the following integrals, identify the method to use to integrate. Choices: Easy integral, substitution, integration by parts.

(a) \( \int x^3 - 3x \, dx \)
   Solution: Easy integral, polynomial!

(b) \( \int e^{4x} \, dx \)
   Solution: Substitution

(c) \( \int \ln x \, dx \)
   Solution: Integration by parts

(d) \( \int x \ln x \, dx \)
   Solution: Integration by parts

(e) \( \int \frac{\ln x}{x} \, dx \)
   Solution: Substitution

(f) \( \int x^5 e^{2x} \, dx \)
   Solution: Integration by parts

(g) \( \int e^{x^2} \, dx \)
   Solution: Can’t integrate it!

(h) \( \int e^x + x^3 \, dx \)
   Solution: Easy integral