Warm-up Problems - January 25, 2006
Solutions

1. An oil company estimates that oil is pumped from one of its oil fields at a rate of

$$R(t) = \frac{50t}{t^2+50} + 5$$

where R is the rate of production (in thousands of barrels per year) and t is the number of years after pumping began.

(a) Find $\int_1^{10} R(t)\, dt$
Solution: This is a u-substitution, using $u = t^2 + 50$, $du = 2t\, dt$.

$$\begin{aligned}
\int_1^{10} \frac{50t}{t^2+50} + 5\ dt &= \int_1^{10} \frac{50t}{t^2+50}\ dt + \int_1^{10} 5\ dt \\
&= \int \frac{25}{u}\ dt + 5t = [25\ln|u| + 5t]_{t=1}^{t=10} \\
&= \left[25\ln|t^2+50| + 5t\right]_{t=1}^{t=10} \\
&= (25\ln|150| + 50) - (25\ln 51 + 5) \\
&= 25\ln\left(\frac{50}{17}\right) + 45 \approx 71.97
\end{aligned}$$

(b) What does your answer in the previous part mean?
Solution: This is the amount of oil pumped from year 1 to year 10

Lecture Problems

2. Researcher at Anheuser-Busch have determined that the length of time a case of Budweiser beer sits on a shelf is a continuous random variable with probability density function

$$f(t) = \begin{cases} \frac{200}{(x+10)^3} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where t is in number of days. You don't need to check that f is a pdf, it is. But, notice that $f(x) \geq 0$ for all x.

(a) Find the probability that a randomly selected case of beer sits on the shelf for between at most 10 days.
Solution:

$$P(0 \leq X \leq 10) = \int_0^{10} f(x)\, dx = \left[-\frac{100}{(t+10)^2}\right]_0^{10} = \frac{3}{4}$$

(b) Find the probability that a randomly selected case of beer sits on the shelf for between at least 10 days.
Solution:

$$\begin{aligned} P(10 \leq X) =& 1 - P(0 \leq X \leq 10) = 1 - \int_{10}^{\infty} f(x)\, dx \\ =& 1 - \left[-\frac{100}{(t+10)^2}\right]_0^{10} = 1 - \frac{3}{4} = \frac{1}{4} \end{aligned}$$

3. Consider an income stream with rate of flow $f(t) = 200e^{0.02t}$.

(a) Find the total income of the stream over 10 years. Explain why your answer is reasonable.
Solution:

$$TI = \int_0^{10} 200e^{0.02t}\, dt = \left[10000e^{0.02t}\right]_0^{10} \approx 2214.03$$

(b) If the interest rate is 5%, compounded continously, find the future value of the income stream in 10 years.
Solution:

$$\begin{aligned} TI =& e^{rT} \int_0^T f(t)e^{-rt}\, dt = e^{(0.05)(10)} \int_0^{10} 200e^{0.02t}e^{-0.05t}\, dt \\ =& e^{0.5} \int_0^{10} 200e^{-0.03t}\, dt = e^{0.5}\left[-\frac{200}{0.03}e^{-0.03t}\right]_0^{10} \approx 2848.79 \end{aligned}$$