

Warm-up Problems - January 23, 2006

Solutions

1. Graph the functions $y = x^2 - 1$ and $y = -x + 1$. Determine the points of intersection of these curves.

Solution: To find the intersection points, solve

$$\begin{aligned}x^2 - 1 &= -x + 1 \\x^2 + x - 2 &= 0 \\(x - 1)(x + 2) &= 0 \\x &= -2, 1\end{aligned}$$

2. Compute the integrals:

(a) $\int_0^1 x \, dx$

Solution:

$$\int_0^1 x \, dx = \left. \frac{x^2}{2} \right|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

(b) $2 \int_0^1 x - x^2 \, dx$

Solution:

$$2 \int_0^1 x - x^2 \, dx = 2 \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = 2 \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{1}{3}$$

(c) $2 \int_0^1 x - x^3 \, dx$

Solution:

$$2 \int_0^1 x - x^3 \, dx = 2 \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = 2 \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{1}{2}$$

Lecture Problems

3. A certain country has a Lorenz curve $f(x) = x^{2.5}$. An economist proposes sweeping reform to provide a more equitable distribution of income. This economist projects that his reforms will result in a Lorenz curve of $g(x) = 0.4x + 0.6x^7$.

- (a) Are these valid Lorenz curves?

Solution: Yes, $f(0) = g(0) = 0$, $f(1) = g(1) = 1$ and each curve is increasing.

- (b) Compute the Gini index for each curve.

Solution: Gini Index for $f(x)$:

$$2 \int_0^1 x - x^{2.5} dx = \frac{3}{7} \approx 0.42857$$

Gini Index for $g(x)$:

$$2 \int_0^1 x - (0.4x + 0.6x^7) dx = \frac{9}{20} = 0.45$$

- (c) Is the economist correct?

Solution: NO! If the economist's projected Lorenz curve is correct, income distribution will be more unequal!