

Warm-up Problems - February 8, 2006

Solutions

1. Find all second order partials (what does this mean???) for the functions below.

(a) $f(x, y) = x^3 + y^3 - 4xy$

Solution:

$$f_x = 3x^2 - 4y$$

$$f_y = 3y^2 - 4x$$

$$f_{xx} = 6x$$

$$f_{yy} = 6y$$

$$f_{xy} = -4$$

(b) $g(x, y, z) = \ln(xy + z + 1)$

Solution:

$$f_x = \frac{y}{xy + z + 1}$$

$$f_y = \frac{x}{xy + z + 1}$$

$$f_z = \frac{1}{xy + z + 1}$$

$$f_{xx} = -\frac{y^2}{(xy + z + 1)^2}$$

$$f_{yy} = -\frac{x^2}{(xy + z + 1)^2}$$

$$f_{zz} = -\frac{1}{(xy + z + 1)^2}$$

$$f_{xy} = f_{yx} = \frac{z + 1}{(xy + z + 1)^2}$$

$$f_{xz} = f_{zx} = -\frac{y}{(xy + z + 1)^2}$$

$$f_{yz} = f_{zy} = -\frac{x}{(xy + z + 1)^2}$$

Lecture Problems

2. Using the function in question 1a, find all critical points, find D and classify the critical points as maximums, minimums or neither.

Solution:

$$3x^2 - 4y = 0 \implies y = \frac{3}{4}x^2$$

$$3y^2 - 4x = 0 \implies 3\left(\frac{3}{4}x^2\right)^2 - 4x = 0 \implies \frac{x}{16}(27x^3 - 64) = 0$$

$$x = 0, \quad x^3 = \frac{64}{27}$$

$$x = 0, x = \frac{4}{3}$$

$$D = 36xy - 16 = 4(9xy - 4)$$

C.P	D	f_{xx}	Type	f
$(0, 0)$	-16	0	Saddle	0
$\left(\frac{4}{3}, \frac{4}{3}\right)$	48	8	Min	$-\frac{64}{27}$