

Warm-up Problems - February 6, 2006

Solutions

1. Find the area between the x -axis and $y = \ln x$ for $1 \leq x \leq e$. **Solution:** You have to integrate by parts:

$$\int_1^e \ln x \, dx = [x \ln x - x]_1^e = [e(1) - e] - [0 - 1] = 1$$

2. For an income stream, rank the following in order of smaller to largest.
- (a) Future value of the income stream
 - (b) Present value of the income stream

Solution: We certainly have $PV < FV$

3. A random variable, X , has pdf:'

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{2}{(x+1)^3} & \text{if } x \geq 0 \end{cases}$$

Find the probability that $X < 0.5$.

Solution:

$$P(X < 0.5) = \int_0^{0.5} \frac{2}{(x+1)^3} \, dx = \left[-\frac{1}{(x+1)^2} \right]_0^{0.5} = \frac{5}{9}$$

Lecture Problems

4. For the functions below, find all critical points ($f_x = 0, f_y = 0$).
- (a) $f(x, y) = x^2 - y^2 + 2x + 6y - 4$
Solution: $(-1, 3)$
 - (b) $f(x, y) = 16xy - x^4 - 2y^2$
Solution: $(0, 0), (4, 16), (-4, 16)$