Warm-up Problems - February 17, 2006
Solutions

1. Find the least squares line, \( y = ax + b \) for the data below.
Using your least squares line, predict a value for \( y(5) \).

\[(\text{a})\]

\[
\begin{array}{c|c|c|c}
\hline
x & y & ax + b & \text{Residual} \\
\hline
1 & 2 & a + b & 2 - a - b \\
2 & 4 & 2a + b & 6 - 2a - b \\
4 & 9 & 4a + b & 9 - 4a - b \\
\hline
\end{array}
\]

Solution: Set up the chart

\[
F(a, b) = (2 - a - b)^2 + (6 - 2a - b)^2 + (9 - 4a - b)^2
= 21a^2 + 3b^2 + 14ab - 100a - 34b + 121
\]

Now, minimize \( F(a, b) \) and you find \( a = \frac{31}{14} \) and \( b = \frac{1}{2} \):

\[
y = \frac{31}{14} x + \frac{1}{2}
\]

To find a prediction for \( y(5) \), we just plug 5 into our least squares line:

\[
y(5) = \frac{31}{14} (5) + \frac{1}{2} = \frac{81}{7}
\]
(b)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>-6</td>
<td>-9</td>
</tr>
</tbody>
</table>

Solution:

\[ y = -\frac{47}{14}x + \frac{7}{2} \]

\[ y(5) = -\frac{47}{14}(5) + \frac{7}{2} = -\frac{93}{7} \]
Lecture Problems

2. Find the least squares line, \( y = ax + b \) for the data below. Using your least squares line, predict \( y(10) \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>xy</th>
<th>x²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>24</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>70</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>120</td>
<td>36</td>
</tr>
<tr>
<td>Σ</td>
<td>24</td>
<td>64</td>
<td>300</td>
</tr>
</tbody>
</table>

**Solution:** This gives, using the formulas

\[
a = \frac{6(300) - (24)(64)}{6(108) - (24)^2} = \frac{11}{3}
\]

\[
b = \frac{64 - \frac{11}{3}(24)}{6} = -4
\]

\[
y = \frac{11}{3}x - 4
\]

\[
y(10) = \frac{98}{3}
\]