

Warm-up Problems - February 17, 2006 Solutions

1. Find the least squares line, $y = ax + b$ for the data below.
Using your least squares line, predict a value for $y(5)$.

(a)

x	1	2	4
y	2	6	9

Solution: Set up the chart

x	y	$ax + b$	Residual
1	2	$a + b$	$2 - a - b$
2	4	$2a + b$	$6 - 2a - b$
4	9	$4a + b$	$9 - 4a - b$

Now let $F(a, b)$ be the sum of these residuals, squared:

$$\begin{aligned} F(a, b) &= (2 - a - b)^2 + (6 - 2a - b)^2 + (9 - 4a - b)^2 \\ &= 21a^2 + 3b^2 + 14ab - 100a - 34b + 121 \end{aligned}$$

Now, minimize $F(a, b)$ and you find $a = \frac{31}{14}$ and $b = \frac{1}{2}$:

$$y = \frac{31}{14}x + \frac{1}{2}$$

To find a prediction for $y(5)$, we just plug 5 into our least squares line:

$$y(5) = \frac{31}{14}(5) + \frac{1}{2} = \frac{81}{7}$$

(b)

$$\begin{array}{c|c|c|c} \mathbf{x} & 1 & 2 & 4 \\ \hline \mathbf{y} & 2 & -6 & -9 \end{array}$$

Solution:

$$y = -\frac{47}{14}x + \frac{7}{2}$$

$$y(5) = -\frac{47}{14}(5) + \frac{7}{2} = -\frac{93}{7}$$

Lecture Problems

2. Find the least squares line, $y = ax + b$ for the data below. Using your least squares line, predict $y(10)$.

	x	y	xy	x²
	2	4	8	4
	3	6	18	9
	3	8	24	9
	5	12	60	25
	5	14	70	25
	6	20	120	36
Σ	24	64	300	108

Solution: This gives, using the formulas

$$a = \frac{6(300) - (24)(64)}{6(108) - (24)^2} = \frac{11}{3}$$

$$b = \frac{64 - \frac{11}{3}(24)}{6} = -4$$

$$y = \frac{11}{3}x - 4$$

$$y(10) = \frac{98}{3}$$