

Warm-up Problems - February 10, 2006

Solutions

1. Solve the following systems of equation for x , y and λ

(a)

$$\begin{aligned}16x + \lambda &= 0 \\ 28y + \lambda &= 0 \\ x + y - 60 &= \lambda\end{aligned}$$

Hint: first solve the first two equations for λ then set these equal. Then use the last equation.

Solution: Solving gives

$$\begin{aligned}x &= \frac{240}{11} \\ y &= \frac{420}{11} \\ \lambda &= \frac{6720}{11}\end{aligned}$$

(b)

$$\begin{aligned}1.6x^{-0.8}y^{0.8} + 10\lambda &= 0 \\ 6.4x^{0.2}y^{-0.2} + 50\lambda &= 0 \\ 10x + 50y - 500000 &= 0\end{aligned}$$

Hint: first solve the first two equations for λ then set these equal. Then use the last equation.

Solution:

$$\begin{aligned}x &= 8000 \\ y &= 10000 \\ \lambda &\approx 0.133842\end{aligned}$$

Lecture Problems

2. For the problem below, determine the function to maximize, the constraint and then set up the function $F(x, y, \lambda)$. If you have time, find the maximum N .

A firm has budgeted \$50,000 per month for labor and materials. If \$ x thousand is spend on labor and \$ y thousand on materials, and if the monthly output (in units) is

$$N(x, y) = 2xy - 4x$$

how should the \$50,000 be allocated to labor and materials to maximize N ?

Solution:

Maximize: $N = 2xy - 4x$.

Constraint: $x + y = 50$

$F = 2xy - 4x + \lambda(x + y - 50)$

Solution: $x = 24, y = 26$, maximum N is 1152.