Warm-up Problems - April 7, 2006
Solutions

NO WARM UP PROBLEMS

Lecture Problems

1. For each of the functions below, sketch a graph and determine if the function is a PDF. If it is not a PDF, explain why.

(a)

$$f(x) = x$$

Solution: Not a pdf. This function is not positive everywhere.

(b)

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Solution: Not a pdf. The total integral is not 1.

(c)

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Solution: Not a pdf. The total integral is not 1.

(d)

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Solution: This is a pdf!

(e)

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Solution: This is a pdf!

(f)

$$f(x) = \begin{cases} \frac{1}{10} & \text{if } 5 \leq x \leq 15 \\ 0 & \text{otherwise} \end{cases}$$

Solution: This is a pdf!

(g)

$$f(x) = \begin{cases} \frac{1}{10} & \text{if } 5 \leq x \leq 20 \\ 0 & \text{otherwise} \end{cases}$$

Solution: This is not a pdf, the total integral is not 1.

2. The daily demand for gasoline (in millions of gallons) in a certain city is a continuous random variable with probability density function

$$f(x) = \begin{cases} 0.15 - 0.01x & \text{if } 0 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is represented by the integral below:

$$\int_0^2 f(x)\,dx$$

Solution: This is the probability that the demand will be between 0 and 2 million gallons.

(b) What is represented by the integral below:

$$\int_2^{12} f(x)\,dx$$

Solution: This is the probability that the demand will be between 2 and 12 million gallons.

(c) What is the probability that the demand will be at most 3 millon gallons?

Solution:

$$P(0 \leq X \leq 3) = \int_0^3 0.15 - 0.01x\,dx = 0.405$$

(d) What is the probability that the demand will be between 4 and 8 millon gallons?

Solution:

$$P(4 \leq X \leq 8) = \int_4^8 0.15 - 0.01x\,dx = 0..36$$

(e) What is the probability that the demand will be between 15 and 20 millon gallons?

Solution:

$$P(15 \leq X \leq 20) = \int_{15}^{20} f(x)\,dx = \int_{15}^{20} 0\,dx = 0$$

(f) What is the probability that the demand will be between 8 and 20 millon gallons?

Solution:

$$P(8 \leq X \leq 20) = \int_8^{20} f(x)\,dx = \int_8^{10} 0.15 - 0.01x\,dx + \int_{10}^{20} 0\,dx = 0.12 + 0$$

3. The time in months that you wait in line at Holmes lounge is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{3}e^{-x/3} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is represented by the integral below:

$$\int_5^8 f(x)\,dx$$

Solution: This is the probability that you will wait between 5 and 8 minutes.

(b) What is represented by the integral below:

$$\int_1^\infty f(x)\,dx$$

Solution: This is the probability that you will wait at least 1 minute.

(c) What is the probability that you will wait 2 minutes?

Solution:

$$P(0 \leq X \leq 2) = \int_0^2 \frac{1}{3}e^{-x/3}\,dx = 1 - e^{-2/3} \approx 0.487$$

(d) What is the probability that you wait more than 2 minutes?

Solution:

$$P(X > 2) = \int_2^\infty \frac{1}{3}e^{-x/3}\,dx = 1 - \int_0^2 \frac{1}{3}e^{-x/3}\,dx = e^{-2/3} \approx .513$$