April 28, 2006 — Final Exam Review Problems

1. Find the Taylor series for \( f(x) = x^3 - e^{2x} \) at \( x = 0 \)

2. Find all Taylor polynomials of degree less than and equal to 5 for \( f(x) = x^7 - 4x^6 + 3x^4 + 2x^3 - x^2 + 5x - 31 \) at \( x = 0 \)

3. Find all Taylor polynomials of degree less than and equal to 5 for \( f(x) = x^7 - 4x^6 + 3x^4 + 2x^3 - x^2 + 5x - 31 \) at \( x = 1 \)

4. Find all Taylor polynomials of degree less than and equal to 5 for \( f(x) = x^7 - 4x^6 + 3x^4 + 2x^3 - x^2 + 5x - 31 \) at \( x = -3 \)

5. Use a degree 4 polynomial to approximate the integrals below. Estimate the error of your approximation.

\[
\int_0^{1/2} e^{-x^2} \, dx
\]

6. Use a degree 4 polynomial to approximate the integrals below. Estimate the error of your approximation.

\[
\int_0^{1/2} \ln(1 + x^2) \, dx
\]

7. Approximate the integral below to within 0.0001. What is the degree of the Taylor polynomial that you used.

\[
\int_0^{1/3} \frac{1}{1 + x^2} \, dx
\]

8. Approximate the integral below to within 0.0001. What is the degree of the Taylor polynomial that you used.

\[
\int_0^{1/3} \frac{1}{1 + x^4} \, dx
\]

9. An income stream of \$1,000e^{-0.05t}\ per year for the next 5 years is invested as it arrives at a compound interest rate of 4%. What is the total income, future value and present value of this income stream?

10. An income stream of \$1,000e^{-0.5t}\ per year for the next 5 years is invested as it arrives at a compound interest rate of 5%. What is the total income, future value and present value of this income stream?

11. Oil is produced from a well at a rate of

\[
R(t) = \frac{100}{t+10} + 10
\]

in millions of barrels of oil per year. Find the total amount of oil produced in years \( t = 1 \) to \( t = 4 \).
12. Oil is produced from a well at a rate of

\[ R(t) = \frac{100t}{t^2 + 10} + 10 \]

in millions of barrels of oil per year. Find the total amount of oil produced in years \( t = 1 \) to \( t = 4 \).

13. For the function, find all critical points. Use the second derivative test to test critical points. Find all maximum and minimum.

\[ f(x, y) = 2x^2 - 2x^2y + 6y^3; \]

14. Find the integral

\[
\int \int_{R} xe^{x+xy} \, dA
\]

where \( R \) is the rectangle \( R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\} \)

15. A new product was introduced one year ago and current sales are $2 million per year. The sales grow at a rate proportional to the difference between sales and the projected maximum of $8 million.

Find the differential equation and find the sale one year from now.

16. Find the solution to the initial value problem:

\[
2xy' + y = 4x^{3/4}
\]

\[ y(1) = 2 \]

17. Find the solution to the initial value problem:

\[
2xy' + y = 4x^{3/4}
\]

\[ y(1) = 4 \]

18. Find the average value of the function \( f(x, y) = x + \frac{1}{xy} \) on the rectangle \( R = \{(x, y) : 2 \leq x \leq 5, 1 \leq y \leq 3\} \).