

April 28, 2006 — Final Exam Review Problems

1. Find the Taylor series for $f(x) = x^3 - e^{2x}$ at $x = 0$
2. Find all Taylor polynomials of degree less than and equal to 5 for $f(x) = x^7 - 4x^6 + 3x^4 + 2x^3 - x^2 + 5x - 31$ at $x = 0$
3. Find all Taylor polynomials of degree less than and equal to 5 for $f(x) = x^7 - 4x^6 + 3x^4 + 2x^3 - x^2 + 5x - 31$ at $x = 1$
4. Find all Taylor polynomials of degree less than and equal to 5 for $f(x) = x^7 - 4x^6 + 3x^4 + 2x^3 - x^2 + 5x - 31$ at $x = -3$
5. Use a degree 4 polynomial to approximate the integrals below. Estimate the error of your approximation.

$$\int_0^{1/2} e^{-x^2} dx$$

6. Use a degree 4 polynomial to approximate the integrals below. Estimate the error of your approximation.

$$\int_0^{1/2} \ln(1 + x^2) dx$$

7. Approximate the integral below to within 0.0001. What is the degree of the Taylor polynomial that you used.

$$\int_0^{1/3} \frac{1}{1 + x^2} dx$$

8. Approximate the integral below to within 0.0001. What is the degree of the Taylor polynomial that you used.

$$\int_0^{1/3} \frac{1}{1 + x^4} dx$$

9. An income stream of $\$1,000e^{-0.05t}$ per year for the next 5 years is invested as it arrives at a compound interest rate of 4%. What is the total income, future value and present value of this income stream?
10. An income stream of $\$1,000e^{-0.5t}$ per year for the next 5 years is invested as it arrives at a compound interest rate of 5%. What is the total income, future value and present value of this income stream?
11. Oil is produced from a well at a rate of

$$R(t) = \frac{100}{t + 10} + 10$$

in millions of barrels of oil per year. Find the total amount of oil produced in years $t = 1$ to $t = 4$.

12. Oil is produced from a well at a rate of

$$R(t) = \frac{100t}{t^2 + 10} + 10$$

in millions of barrels of oil per year. Find the total amount of oil produced in years $t = 1$ to $t = 4$.

13. For the function, find all critical points. Use the second derivative test to test critical points. Find all maximum and minimum.

$$f(x, y) = 2x^2 - 2x^2y + 6y^3;$$

14. Find the integral

$$\iint_R x e^{x+xy} dA$$

where R is the rectangle $R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$

15. A new product was introduced one year ago and current sales are \$2 million per year. The sales grow at a rate proportional to the difference between sales and the projected maximum of \$8 million.

Find the differential equation and find the sale one year from now.

16. Find the solution to the initial value problem:

$$\begin{aligned} 2xy' + y &= 4x^{3/4} \\ y(1) &= 2 \end{aligned}$$

17. Find the solution to the initial value problem:

$$\begin{aligned} 2xy' + y &= 4x^{3/4} \\ y(1) &= 4 \end{aligned}$$

18. Find the average value of the function $f(x, y) = x + \frac{1}{xy}$ on the rectangle $R = \{(x, y) : 2 \leq x \leq 5, 1 \leq y \leq 3\}$.