Warm-up Problems - April 19, 2006
Solutions

1. Your bus arrives at the bus stop every 10 minutes, but you never remember to check the bus schedule.

   (Try to think about this problem without considering probability density functions.)

   (a) What is the probability that that when you get to the stop, you have to wait at 1 minutes or less?

   Solution:

   $$P(X < 2) = \frac{1}{10}$$

   (b) What is the probability that that when you get to the stop, you have to wait at 9 minutes or less?

   Solution:

   $$P(X < 2) = \frac{9}{10}$$

   (c) If you go to the bus stop for 10 days, what would you expect your average wait time to be?

   Solution:

   $$E(X) = 5$$

2. Consider a group of patients that have been treated for an acute disease, such as cancer. Let X be the number of years a person lives after receiving the treatment (X is the "survival time"). The continuous random variable, X, will have an exponential probability density function. In other words, the pdf and cdf will be:

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{\lambda} e^{-x/\lambda} & \text{if } x \geq 0 \end{cases} \qquad F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x/\lambda} & \text{if } x \geq 0 \end{cases}$$

   (a) If the average survival time is 10 years, find λ.
   Solution: We know that $\mu = \lambda$, so this would mean $\lambda = 10$.

   (b) If the median survival time is 10 years, find λ.
   Solution: We know that $m = \lambda \ln 2$, so this would mean $\lambda = \frac{10}{\ln 2}$.

   (c) If the probability that a patient survives 5 years is 90%, find λ.
   Solution: In this case we know that $P(X \geq 5) = 0.9$ so we solve:

$$\begin{aligned} 1 - F(5) &= 0.9 \\ 1 - \left(1 - e^{-5/\lambda}\right) &= 0.9 \\ e^{-5/\lambda} &= 0.9 \\ -5/\lambda &= \ln 0.9 \\ \lambda &= -\frac{5}{\ln 0.9} \approx 47.46 \end{aligned}$$

Lecture Problems

3. Find the probabilities for the standard normal distribution.

   (a) $P(0 \leq X \leq 1.26) = 0.3962$

   (b) $P(X \geq 1.26) = 0.5 - 0.3962 = 0.1038$

   (c) $P(1.190 \leq X \leq 1.26) = 0.3962 - 0.3830 = 0.0132$

   (d) $P(-1.21 \leq X \leq 0) = 0.3869$

   (e) $P(X \leq -1.21) = 0.5 - 0.3869 = 0.1131$

   (f) $P(-0.54 \leq X \leq 1.21) = 0.3869 + 0.2054 = 0.5923$

   (g) $P(-1.21 \leq X \leq 0.54) = 0.2054 + 0.3869 = 0.5923$

Area Under the Standard Normal Curve ($\mu = 0$, $\sigma = 1$) From 0 to z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000