

## Warm-up Problems - April 12, 2006

### Solutions

1. For the following functions:

- Check that the functions are PDF's.
- Find the CDF.
- Using the CDF, find  $P(3/4 < X < 11/4)$ .

(a)

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x > 1 \end{cases} \quad F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

$$P\left(\frac{3}{4} < X < \frac{11}{4}\right) = F\left(\frac{11}{4}\right) - F\left(\frac{3}{4}\right) = 1 - \frac{3}{4} = \frac{1}{4}$$

(b)

$$f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } 2 \leq x \leq 3 \\ 0 & \text{if } x > 3 \end{cases} \quad F(x) = \begin{cases} 0 & \text{if } x < 2 \\ x - 2 & \text{if } 2 \leq x \leq 3 \\ 1 & \text{if } x > 3 \end{cases}$$

$$P\left(\frac{3}{4} < X < \frac{11}{4}\right) = F\left(\frac{11}{4}\right) - F\left(\frac{3}{4}\right) = \left(\frac{11}{4} - 2\right) - 0 = \frac{3}{4}$$

## Lecture Problems

2. Find the mean, or expected value of each of the PDF's below.

(a)

$$f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } 2 \leq x \leq 3 \\ 0 & \text{if } x > 3 \end{cases}$$

**Solution:**  $\mu = \frac{5}{2}$

(b)

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{3}{2}(1 - x^2) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$$

**Solution:**  $\mu = \frac{3}{8}$