Warm-up Problems - April 10, 2006
Solutions

1. For the probability density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) $P(0 < X < 1) = \frac{1}{8} = 0.125$

   (b) $P(X > 1) = 1 - P(X < 1) = 1 - \frac{1}{8} = \frac{7}{8}$

   (c) Find the value m such that $P(X > m) = \frac{1}{2}$.
   (This is called the median of the distribution.)
   Solution: We need to solve the equation

$$\int_0^m f(x) = \frac{1}{2}$$

   This gives us

$$\frac{m^3}{8} = \frac{1}{2}$$

   Solving gives $m = 4^{1/3} \approx 1.587$

2. For the probability density function

$$f(x) = \begin{cases} \frac{1}{10}e^{-x/10} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) $P(0 < X < 1) = 1 - e^{-1/10} \approx 0.095$

   (b) $P(X > 1) = 1 - P(X < 1) = e^{-1/10} \approx 0.905$

   (c) Find the value m such that $P(X > m) = \frac{1}{2}$.
   (This is called the median of the distribution.)
   Solution: We need to solve the equation

$$\int_0^m f(x) = \frac{1}{2}$$

   This gives us

$$1 - e^{-m/10} = \frac{1}{2}$$

   Solving gives $m = 10 \ln 2 \approx 6.93$.

Lecture Problems

3. A continuous random variable has cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4}x^2 & \text{if } 0 \le x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$$

Use this to find:

(a) $P(X < 1) = F(1) = \frac{1}{4}$

(b) $P(X > 1) = 1 - P(X < 1) = 1 - F(1) = \frac{3}{4}$

(c) $P(X < 3/4) = F(3/4) = \frac{9}{64}$

(d) $P(X > 3/4) = 1 - P(X < 3/4) = 1 - F(3/4) = \frac{55}{64}$

(e) $P(1/2 < X < 1) = F(1) - F(1/2) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

(f) Find $f(x)$, the probability density function.

Solution: Since $F'(x) = f(x)$ we must have

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

4. Repeat Problem 3 for the cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{9}x^2 & \text{if } 0 \leq x \leq 3 \\ 1 & \text{if } x > 2 \end{cases}$$

(a)

$$P(X < 1) = F(1) = \frac{1}{9}$$

(b)

$$P(X > 1) = 1 - P(X < 1) = 1 - F(1) = \frac{8}{9}$$

(c)

$$P(X < 3/4) = F(3/4) = \frac{1}{16}$$

(d)

$$P(X > 3/4) = 1 - P(X < 3/4) = 1 - F(3/4) = \frac{15}{16}$$

(e)

$$P(1/2 < X < 1) = F(1) - F(1/2) = \frac{1}{9} - \frac{1}{36} = \frac{1}{12}$$

(f) Find $f(x)$, the probability density function.
Solution: Since $F'(x) = f(x)$ we must have

$$f(x) = \begin{cases} \frac{2}{9}x & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

5. Give the pdf's below, the find corresponding cdf.

(a)

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases} \qquad F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 1 & \text{if } x > 2 \end{cases}$$

(b)

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \qquad F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x - \frac{1}{2} & \text{if } 1 \leq x \leq 3 \\ 1 & \text{if } x > 3 \end{cases}$$