Solutions

1. For the probability density function

\[ f(x) = \begin{cases} \frac{3}{8}x^2 & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases} \]

(a) \( P(0 < X < 1) = \frac{1}{8} = 0.125 \)

(b) \( P(X > 1) = 1 - P(X < 1) = 1 - \frac{1}{8} = \frac{7}{8} \)

(c) Find the value \( m \) such that \( P(X > m) = \frac{1}{2} \).

(This is called the median of the distribution.)

Solution: We need to solve the equation

\[ \int_0^m f(x) = \frac{1}{2} \]

This gives us

\[ \frac{m^3}{8} = \frac{1}{2} \]

Solving gives \( m = 4^{1/3} \approx 1.587 \)

2. For the probability density function

\[ f(x) = \begin{cases} \frac{1}{10}e^{-x/10} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

(a) \( P(0 < X < 1) = 1 - e^{-1/10} \approx 0.095 \)

(b) \( P(X > 1) = 1 - P(X < 1) = e^{-1/10} \approx 0.905 \)

(c) Find the value \( m \) such that \( P(X > m) = \frac{1}{2} \).

(This is called the median of the distribution.)

Solution: We need to solve the equation

\[ \int_0^m f(x) = \frac{1}{2} \]

This gives us

\[ 1 - e^{-m/10} = \frac{1}{2} \]

Solving gives \( m = 10 \ln 2 \approx 6.93 \).
Lecture Problems

3. A continuous random variable has cumulative distribution function:

\[ F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{1}{4}x^2 & \text{if } 0 \leq x \leq 2 \\
1 & \text{if } x > 2 
\end{cases} \]

Use this to find:

(a) \( P(X < 1) = F(1) = \frac{1}{4} \)

(b) \( P(X > 1) = 1 - P(X < 1) = 1 - F(1) = \frac{3}{4} \)

(c) \( P(X < 3/4) = F(3/4) = \frac{9}{64} \)

(d) \( P(X > 3/4) = 1 - P(X < 3/4) = 1 - F(3/4) = \frac{55}{64} \)

(e) \( P(1/2 < X < 1) = F(1) - F(1/2) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16} \)

(f) Find \( f(x) \), the probability density function.

**Solution:** Since \( F'(x) = f(x) \) we must have

\[ f(x) = \begin{cases} 
\frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\
0 & \text{otherwise} 
\end{cases} \]
4. Repeat Problem 3 for the cumulative distribution function:

\[ F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{1}{3}x^2 & \text{if } 0 \leq x \leq 3 \\
1 & \text{if } x > 2 
\end{cases} \]

(a) \[ P(X < 1) = F(1) = \frac{1}{9} \]

(b) \[ P(X > 1) = 1 - P(X < 1) = 1 - F(1) = \frac{8}{9} \]

(c) \[ P(X < 3/4) = F(3/4) = \frac{1}{16} \]

(d) \[ P(X > 3/4) = 1 - P(X < 3/4) = 1 - F(3/4) = \frac{15}{16} \]

(e) \[ P(1/2 < X < 1) = F(1) - F(1/2) = \frac{1}{9} - \frac{1}{36} = \frac{1}{12} \]

(f) Find \( f(x) \), the probability density function.

Solution: Since \( F'(x) = f(x) \) we must have

\[ f(x) = \begin{cases} 
\frac{2}{9}x & \text{if } 0 \leq x \leq 3 \\
0 & \text{otherwise} 
\end{cases} \]

5. Give the pdf’s below, the find corresponding cdf.

(a) \[ f(x) = \begin{cases} 
\frac{1}{2} & \text{if } 0 \leq x \leq 2 \\
0 & \text{otherwise} 
\end{cases} \]

\[ F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\
1 & \text{if } x > 2 
\end{cases} \]

(b) \[ f(x) = \begin{cases} 
\frac{1}{2} & \text{if } 1 \leq x \leq 3 \\
0 & \text{otherwise} 
\end{cases} \]

\[ F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{1}{2}x - \frac{1}{2} & \text{if } 1 \leq x \leq 3 \\
1 & \text{if } x > 3 
\end{cases} \]