FINAL
Math 2210

- You have 2 hours to complete this exam.
- Show all your work (and make it neat!). If I can't follow your work or it is missing you WILL NOT get credit.
- You can use a sheet of notes.
- If you run out of space, use the back of these sheets.
- Each problem is worth 10 points.

Name: ____

1. Find the extrema of the function

$$f(x,y) = x^2y - 6y^2 - 3x^2$$

2. Evaluate the surface integral:

$$\iint_G \mathbf{F} \bullet \mathbf{n}\, dS$$

where G is the surface $z = \sqrt{9 - x^2 - y^2}$ and $z = 0$ (so the surface is a hemisphere on top and a disc in the xy-plane on the bottom), $\mathbf{n}$ is the outward pointing normal and

$$\mathbf{F} = (\sin x, (1 - \cos x)y, 4z)$$

(Hint: think about different ways you might be able to compute this!)

3. Consider the line integral

$$\oint_C xy\, dx + (x^2 + y^2)\, dy$$

where C is the unit circle traversed in the counter-clockwise direction.

   (a) Compute the line integral directly (as a line integral!).

   (b) Compute the line integral by appying Green's Theorem.

4. Let R be the region in $\mathbb{R}^3$ defined by the following inequalities:

$$z \leq 4 - y^2 \qquad y \geq 0 \qquad z \geq 0 \qquad x \geq z \qquad x + z \leq 9$$

Write the volume of R as an iterated triple integral.
(Don't compute the integral.)

5. Consider the function

$$f(x,y) = x^2y - xy^2 + y^3$$

From the point $(-2, 1)$ and in the direction $(3, 2)$, is f increasing or decreasing and at what rate?

6. Consider the double integral below:

$$\iint_R y - x\, dA$$

where R is the triangle in the xy-plane with vertices $(0,0)$, $(0,-1)$, $(4,2)$. Make the change of varibles below:

$$\begin{aligned} x &= 2v \\ y &= -u + v \end{aligned}$$

(a) Compute the Jacobian of this transformation.

(b) The transformation transforms the triangle R into a triangle in the uv-plane (you don't have to show this). What are the vertices of this triangle in the uv-plane?

(c) Write the double integral as a double integral over the triangle in the uv-plane.

(d) Compute the integral.