Final
Math 2210 - 2

Name: _____

Instructions:

- You have 2 hours to complete this exam.
- Show all your work. If I can't follow your work or it is missing you WILL NOT get credit. (This holds for graphing too!)
- Use the back of these sheets if you run out of room. Make sure I can find your work.
- No calculators, but you are allowed one "cheat sheet."
- Each problem is worth 10 points.

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1. Let C be the curve in $\mathbb{R}^2$ determined by the parametric equation

$$\sigma(t) = (t, t^n) \qquad 0 \leq t \leq 1$$

(where n is some unknown positive integer). Compute the line integral:

$$\int_C y\, dx + (3y^3 - x)\, dy$$

2. Consider the vector field:

$$\mathbf{F}(x, y, z) = \left(\frac{2x}{yz}, \frac{-x^2}{y^2 z}, \frac{-x^2}{yz^2}\right)$$

Is F a conservative vector field? Justify your answer!

3. Consider the conservative vector field (you don't need to check this!):

$$\mathbf{F}(x,y) = \left(\frac{2x}{y^2+1}, -\frac{2y(x^2+1)}{(y^2+1)^2}\right)$$

(a) Find a potential function for $\mathbf{F}$.

(b) Let C be the curve determined by

$$\sigma(t) = (t^3 - 1, t^6 - t) \qquad 0 \leq t \leq 1$$

Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

4. Let C be the curve made up of the four sides of the square:

$$\sigma(t) = \begin{cases} (t, 0) & \text{if } 0 \leq t \leq 2 \\ (2, t-2) & \text{if } 2 < t \leq 4 \\ (6-t, 2) & \text{if } 4 < t \leq 6 \\ (0, 8-t) & \text{if } 6 < t \leq 8 \end{cases}$$

Compute the line integral by applying Green's Theorem

$$\oint_C (x^2 + y^2)\, dx - 2xy\, dy$$

5. Consider the curve defined by

$$\mathbf{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$$

At the point $\mathbf{r}(1) = (1, 1, 2/3)$ (so when $t = 1$), find

(a) $\mathbf{T}$, the unit tangent vector,

(b) $\mathbf{N}$, the unit normal vector,

(c) $\mathbf{B}$, the unit bi-normal vector,

(d) κ, the curvature of the curve.

6. Consider the function

$$f(x, y) = x^3 + y^3 - 6xy$$

Find all local maxima, local minima and saddle points for f.

7. Let R be the region of the xy-plane defined by

$$x \geq 0, \quad y \geq x^2, \quad y \leq 10 - x^2$$

Find the integral:

$$\iint_R xy \, dA$$

8. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 9$ that is trapped between the planes $z = 1$ and $z = 2$.