

TEST 1

Math 111

You have 1 hour to complete the test. You may not use a calculator. Show all relevant work (or else no credit). Use the backs of these pages if necessary.

Name _____

1. (10 points)

(a) What does a function have to satisfy to be continuous at a point?

(b) What does a function have to satisfy to be continuous?

2. (10 points) True or False, circle the correct answer:

T F For all functions f and g , $f \circ g = g \circ f$.

T F For all functions, f , $\lim_{x \rightarrow c} f(x) = f(c)$

3. (10 points) In the equation below, can y be expressed as a function of x ? ($y = f(x)$) If it can be done, find the function. If it cannot, why not?

$$\sin x + 2y + xy + x^2y = 3$$

4. (10 points) Find f and g so that $h = f \circ g$. (Make sure that neither $f(x) = x$ nor $g(x) = x$)

$$h(x) = \sin(x^2 + \cos x)$$

5. (10 points) Find the limit

$$\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

6. (12 points) Find the limits or state that the limit does not exist for the function g graphed below.

(a) $\lim_{x \rightarrow -2} g(x)$

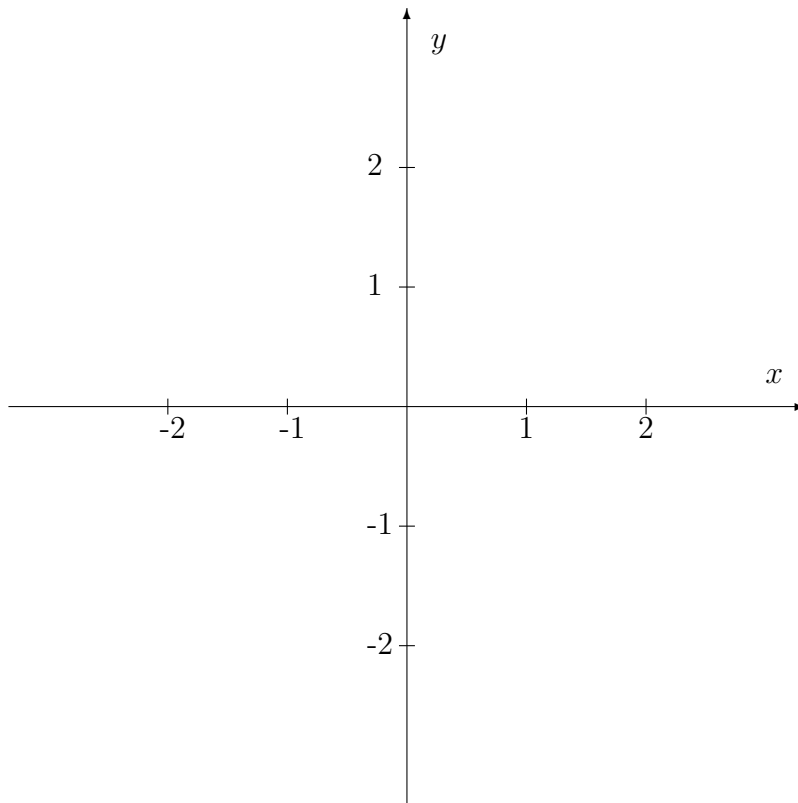
(b) $\lim_{x \rightarrow 1^-} g(x)$

(c) $\lim_{x \rightarrow 1^+} g(x)$

(d) $\lim_{x \rightarrow 1} g(x)$

(e) True or False: g is continuous at 0.

(f) True or False: g is continuous at -2.



7. (9 points) Find the following limits: (Note that $\lceil x \rceil$ is the greatest integer function)

(a) $\lim_{t \rightarrow \frac{1}{2}^+} \lceil 2t \rceil$

(b) $\lim_{t \rightarrow \frac{1}{2}^-} \lceil 2t \rceil$

(c) $\lim_{t \rightarrow \frac{1}{2}} \lceil 2t \rceil$

8. (10 points)

(a) What is the natural domain of f ?

$$f(x) = \frac{x^3 + 8}{x + 2}$$

(b) Can you define f (f as in part ??) at the points not in the domain of f , so that f is continuous everywhere? If so how, if not why not?

9. (10 points) Given the function f below, what do a and b need to be so that f is continuous?

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq 2 \\ ax + b & \text{if } 2 < x < 3 \\ 3x + 1 & \text{if } x \geq 3 \end{cases}$$

10. (10 points)

$$g(x) = \sqrt{2x - 1}$$

(a) Using the limit definition of derivative, find $g'(x)$.

(b) Find the equation of the tangent line at $x = 5$ for g .