WASHINGTON UNIVERSITY MATHEMATICS TEACHER CIRCLE
WHAT IS CHAOS?

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Problems:

1) Suppose that $f : [0,1] \to [0,1]$ and x_0 is a fixed point of f^n, $n \in \mathbb{N}$. Given another positive integer $m \in \mathbb{N}$ under what conditions can x_0 be a fixed point of f^m? For example, if $n = 2$ and x_0 is not a fixed point of f is it possible for x_0 to be a fixed point of f^{71}?
2) Consider the tent map, f, given by
$$f(x) = \begin{cases} 2x, & 0 \le x < \frac{1}{2}, \\ 2(1-x), & \frac{1}{2} \le x \le 1. \end{cases}$$
   a. What are the fixed points of f^3? Of f^4?
   b. Find a cycle of length at least 5.
3) Examine the behavior of the orbits induced by
$$f(x) = \begin{cases} \frac{3}{2}x, & 0 \le x \le \frac{1}{2}, \\ \frac{3}{2}(x - \frac{1}{3}), & \frac{1}{2} < x \le 1. \end{cases}$$
Notice that the only fixed points of f are $x = 0$ and $x = 1$. Can anything be said about the long-term behavior of $f^n(x)$ for $0 < x < 1$?
4) Consider the map $f(x) = x + \sin(2\pi x)$ for all real numbers x. What are the fixed points? Are they repelling or attracting?
5) Construct a map on $[0,1]$ which has both attracting and repelling fixed points.
6) If x_0 is an attracting fixed point of f^2 what can be said about the cycle $x_0 \mapsto f(x_0) \mapsto x_0$ for f?

Date: March 19, 2008.